M54-LE5-005
Mathematics 54 Elementary Analysis II
Fifth Long Exam First Semester, AY 2013 -2014

I. If Ψ is the function defined by $\Psi(x,y)=\sqrt{4-x^2-y^2}+\ln(x-y)$, identify and sketch as a region of the plane the domain of Ψ. *4 points*

II. Consider the function ψ defined by: $\psi(x,y)=\begin{cases}\dfrac{(x-1)^4-y^4}{(x-1)^2+y^2},(x,y)\neq(1,0)\\ 1,(x,y)=(1,0)\end{cases}$. Determine the points where the function is discontinuous, if there are any. Classify each discontinuity as either removable or essential. *4 points*

III. Consider the function *f* defined by $f(\theta,s)=\dfrac{\cos 3\theta}{s}$.

1. Using only the definition, evaluate $f_s(\pi,4)$. *4 points*
2. Suppose a certain function *g* is continuous on some open disk *D* containing $(\pi,4)$ satisfies $g_s(\theta,s)=f(\theta,s)$ for all points (θ,s) in *D*. Find $g_{\theta s\theta}(\pi,4)$. *3 points*

IV. If *z* is a function of *x* and *y* implicitly defined by $y^3\tan(xz)=z^2-e^{xy}$, evaluate $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$ at the point (0,2,-1). *5 points*

V. Let S be the graph of the function *h* defined by $h(x,y)=\sin(e^{4x}-y^2)$.

1. Find an equation of the tangent plane to *S* at the point where $x = 0$ and $y = 1$. *5 points*
2. Using the tangent plane in the previous item, approximate $\sin(e^{0.04}-(0.99)^2)$. *3 points*
3. If, in addition, *x* and *y* are functions of *u* and *v* given by: $x=\ln(u^2v-7)$ and $y=\dfrac{u^3-2v}{4}$, find $\dfrac{\partial h}{\partial u}$ at the point where $u = v = 2$, using Chain Rule. *4 points*

VI. Show that the following limit does not exist: $\lim\limits_{(x,y)\to(0,0)}\dfrac{x^2y^6}{x^4+2y^{12}}$. *4 points*

VII. Erza is to paint scarlet the entire outside of a closed tin can in the shape of a right-circular cylinder with radius 4 inches and height 16 inches. Using differentials, approximate how much scarlet paint Erza would need if she wishes the paint to be 0.02 inch thick. *4 points*

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