M55-LE1-002

Mathematics 55 — Elementary Analysis III

First Long Exam — Second Semester, AY 2014 -2015

1. Let $f(x,y) = y^2e^{2x} + \cos(xy)$ and $P_0 = (0,2)$. *6 points*
   (a) Find the directional derivative of f at the point P_0 in the direction of $\vec{v} = \langle 1,-1 \rangle$.
   (b) Find the maximum rate of change of f at P_0.
   (c) Find a unit vector in the direction in which f decreases most rapidly at P_0.

2. Determine the equations of the normal line to the surface S at the point (0,0,1) if S is described by the Cartesian equation $\ln(2y+z) = xz^2$. *4 points*

3. Let S be the parametric surface defined by $\vec{R}(u,v) = uv^2\hat{i} + (u-v)\hat{j} + u^2\hat{k}$. *4 points*
   (a) Compute $\vec{R}_u \times \vec{R}_v$.
   (b) Find an equation of the tangent plane to S at the point (1,2,1).

4. Find and classify all critical points (x,y) of the function $f(x,y) = x^3 + 5y^3 - 3x^2y - 3y + 1$. *6 points*

5. Use the method of Lagrange multipliers to find the maximum and minimum values of the function $f(x,y,z) = 10x - 8y + 6z$ subject to the constraint $x^2 + y^2 + z^2 = 50$. *4 points*

6. Evaluate $\int_0^{6\sqrt{\pi}} \int_{y/6}^{\sqrt{\pi}} \sin(x^2)\,dx\,dy$. *4 points*

7. Evaluate $\int_0^{\sqrt{2}} \int_x^{\sqrt{4-x^2}} \sqrt{x^2+y^2}\,dy\,dx$ by converting to polar coordinates. *4 points*

8. Set up an iterated double integral that yields the (surface) area of the portion of the plane $2x + 2y + z = 4$ in the first octant enclosed by the yz-plane and the cylinder $y = x^2$. *4 points*

9. Set up an iterated double integral in polar coordinates that gives the mass of a lamina in the shape of the region in the first quadrant inside the circle $x^2 + y^2 = 9$ but outside the circle $x^2 + (y-1)^2 = 1$ having density $\delta(x,y) = x^3y$. *4 points*

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