M55-LE1-003

Mathematics 55 — Elementary Analysis III

First Long Exam — Second Semester, AY 2014 -2015

1. Let f be a differentiable function of two variables with $f(0,4) = 0$ and $\nabla f(0,4) = \langle 2, \sqrt{3} \rangle$.

   a. Find the maximum rate of change of f at $(0,4)$ and the direction in which it occurs. *(2 pts)*

   b. Find the instantaneous rate of change of f at $(0,4)$ in the direction of the vector $\langle -1, \sqrt{3} \rangle$. *(2 pts)*

2. Find an equation for the tangent plane to surface $z = xy + \sin(z)$ at the point $(\pi, 1, \pi)$. *(3 pts)*

3. The function h is continuous on $\mathbb{R}^2$ with partial derivatives

$$h_x(x,y) = x^2 - y \quad \text{and} \quad h_y(x,y) = y - x.$$

   Find the critical points (a,b) of h, and determine whether each $(a,b,h(a,b))$ is a relative maximum point, a relative minimum point, or a saddle point of h. *(6 pts)*

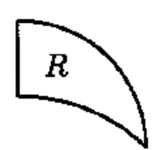
4. Use the method of Lagrange multipliers to find the lowest point on the curve of intersection of the plane $4y - x = 5$ and the paraboloid $z = x^2 + y^2$. *(5 pts)*

5. Evaluate *(4 pts)*

$$\int_0^4 \int_{\sqrt{y}}^2 \sqrt{1 + x^3}\, dx\, dy$$

6. Evaluate by polar coordinates the double integral

$$\iint_R \frac{1}{\sqrt{x^2 + y^2}}\, dA$$



   where R is the region in the first quadrant inside $x^2 + y^2 = 2y$ and outside $x^2 + y^2 = 2$. *(5 pts)*

7. Let R be the triangular region bounded by $y = |x|$ and $y = 1$, and let $G(x,y) = x^2 - y^2 + 1$. Set up the iterated integrals equal to each of the following. (Do not evaluate.)

   a. area of R *(2 pts)*

   b. volume of the solid bounded by the xy plane, the graph of G, and the cylinder with rulings parallel to the z axis whose xy trace is the boundary of R *(3 pts)*

   c. surface area of the graph of G directly above R *(3 pts)*

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