UP SCHOOL OF STATISTICS STUDENT COUNCIL

Education and Research

erho.weebly.com erhomyhero@gmail.com /erhoismyhero @erhomyhero

M55-LE1-004

Mathematics 55 | Elementary Analysis III

First Long Exam | Second Semester, AY 2014 -2015

1. Let $f(x,y) = \dfrac{e^{2y}}{x} - x^3y + 4$. (5 pts.)

   (a) Find the directional derivative of f at the point $(1,0)$ along the direction of the vector $\langle 5, 12\rangle$.

   (b) Find a vector along which f decreases most rapidly at the point $(1,0)$.

2. The surface of revolution S is obtained by revolving the curve $y = e^x$ about the x-axis.

   (a) If S is described by $\vec{R}(x,\theta) = \langle x, e^x\cos\theta, e^x\sin\theta\rangle$, give an equation of the tangent plane to S when $x = 0$ and $\theta = \frac{\pi}{2}$. (4 pts.)

   (b) Show that the Cartesian equation of S is $y^2 + z^2 = e^{2x}$. (1 pt.)

   (c) Using the Cartesian equation of S in (b), find a normal vector to S at the point $(\ln 2, 1, -1)$. (2 pts.).

3. Find all relative extrema and saddle points of $f(x,y) = x^3 + 3xy - 3y^2 + 2$. (6 pts.)

4. Use the method of Lagrange multipliers to find the points on the ellipse $2x^2 + y^2 = 18$ which are closest to and farthest from the point $(2,0)$.
   Hint: Consider $f(x,y) = (\textit{distance from } P(x,y) \textit{ to } (2,0))^2$. (5 pts.)

5. Evaluate the following iterated double integrals. (5 pts. each)

   (a) $\displaystyle\int_0^2 \int_{x^2}^4 xe^{y^2}\, dydx$

   (b) $\displaystyle\int_{-3}^0 \int_0^{\sqrt{9-y^2}} y\sqrt{x^2+y^2}\, dxdy$

6. SET UP an iterated double integral that represents the surface area of the plane $10x + 4y + 2z = 20$ that lies inside the cylinder $x^2 + y^2 = 4$. (3 pts.)

7. SET UP an iterated double integral that gives the volume of the solid in the first octant bounded above by the paraboloid $z = 4 - x^2 - y^2$, below by the xy-plane and on the sides by the planes $x + y = 1$, $x = y$ and $x = 0$. (4 pts.)

- E N D O F E X A M -