UP SCHOOL OF STATISTICS STUDENT COUNCIL
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EDUCATION AND RESEARCH

M55-LE2-002

Mathematics 55 — Elementary Analysis III

Second Long Exam — Second Semester, AY 2014 -2015

1. Use rectangular coordinates to evaluate the triple integral $\iiint_G y\,dV$, where G is the solid bounded by $2x+3y+2z=6$ and the coordinate planes.

2. Use cylindrical coordinates to find the volume of the solid in the first octant bounded by the planes $y = 2z$, $x = 0$, $z = 0$ and the cylinder $x^2+y^2=4$.

3. Use spherical coordinates to set up the iterated integral equal to the mass of the solid bounded above by the sphere $x^2+y^2+(z-2)^2=4$ and bounded below by the cone $z=\sqrt{x^2+y^2}$ if the density at any point (x,y,z) on the solid is $x + 1$.

4. Let $F(x,y,z)=2ye^{2x}\hat{i}+e^{2x}\hat{j}+3z^2\hat{k}$. Show that F is conservative and use a potential function for F to find the value of the line integral $\int_C F dR$ where C is any sectionally smooth curve from the point (ln 2, 1, 1) to the point (ln 2, 2, 2).

5. Evaluate the line integral $\int_C (x^2+xy)dx+(y^2-xy)dy$ where C consists of the line segment $y = x$ from the point (0,0) to the point (2,0) and the vertical line from (2,2) to (2,0).

6. Use Green's Theorem to evaluate the line integral $\oint_C y^2dx+x^2dy$ where C is the closed curve determined by the x-axis, the line $x = 1$ and the curve $y = x^2$ traversed in counter-clockwise direction.

7. Let the surface S be given by: $\vec{r}(u,v)=u\cos v\hat{i}+u\sin v\hat{j}+(v+3)\hat{k}, 0\le u\le 1, 0\le v\le \pi$
   (a) Find $\vec{r}_u\times\vec{r}_v$.
   (b) Evaluate the surface integral $\iint_S \sqrt{x^2+y^2}\,d\sigma$.

8. Let $F(x,y,z)=-x\hat{i}+(y+2)\hat{j}+z\hat{k}$ be the velocity field of a fluid and let S be the portion of the plane $3x+2y+z=6$ in the first octant. Find the flux of F across S.

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