M55-LE2-003

Mathematics 55 — Elementary Analysis III

Second Long Exam — Second Semester, AY 2014 -2015

1. Evaluate using cylindrical coordinates: (4 pts)

$$\int_0^2 \int_0^{\sqrt{4-y^2}} \int_0^{\sqrt{4x^2+4y^2}} \frac{xz}{x^2+y^2}\, dz dx dy$$

2. Set up the iterated triple integral in spherical coordinates that gives the mass of the solid bounded by $x^2+(y-1)^2+z^2=1$, $z=\sqrt{x^2+y^2}$, and $z=0$ if the density at any point is $z-x$. (4 pts)

3. Find the work done by $\vec{F}(x,y,z) = <z, x, -y>$ in moving a particle along the parametric curve $\vec{R}(t) = <t, t^2, 2t>$, $0 \le t \le 4$. (4 pts)

4. Given: $\vec{F}(x,y) = \left\langle e^{2y} - \dfrac{y}{x} - 3x^2, 2xe^{2y} - \ln x \right\rangle$

   (a) Show that $\vec{F}$ is conservative. (2 pts)

   (b) Find all potential functions of $\vec{F}$. (4 pts)

   (c) Use the Fundamental Theorem for Line Integrals to evaluate $\int_C \vec{F} \cdot d\vec{R}$ if C is any path from $(1,0)$ to $(e,1)$. (2 pts)

5. Use Green's Theorem to evaluate $\displaystyle\oint_C (\ln(x^2-1) + xy^2 - 2xy)\, dx + x^2 y\, dy$, where C is the postively-oriented triangle with vertices at (0,0), (3,9), and (3,0). (4 pts)

6. Evaluate $\displaystyle\iint_S \frac{z-y^2}{\sqrt{1+4x^2+16y^2}}\, dS$, where S is the portion of the paraboloid $z = x^2+2y^2$ above the circle $x^2+y^2=1$ in the xy-plane. (4 pts)

7. Given: $\vec{F} = yz\,\hat{i} + 4xy\,\hat{j} + 2y^2\,\hat{k}$

   (a) Determine the divergence and curl of $\vec{F}$. (3 pts)

   (b) Evaluate $\displaystyle\iint_S \vec{F}(x,y,z) \cdot d\vec{S}$, where S is the positively-oriented portion of the plane $2x+y+z=2$ in the first octant. (4 pts)

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