UP SCHOOL OF STATISTICS STUDENT COUNCIL

Education and Research

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EDUCATION AND RESEARCH

M55-LE2-004

Mathematics 55 — Elementary Analysis III

Second Long Exam — Second Semester, AY 2014 -2015

1. Using spherical coordinates, evaluate $\int_0^2 \int_0^{\sqrt{4-y^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{8-x^2+y^2}} \frac{1}{x^2 + y^2 + z^2} dzdxdy$.

2. Setup an iterated integral in cylindrical coordinates that gives the volume of the solid in the first octant bounded by the cylinder $x^2 + y^2 = 1$ and the plane $z = x$.

3. Given the vector field $\vec{F}(x,y,z) = \langle z - x, 2y, 2xz \rangle$
   a. Determine $div\ \vec{F}$ and $curl\ \vec{F}$.
   b. Find the work done by $\vec{F}$ in moving an object along the curve C defined by $\vec{R}(x,y,z) = \langle t^2, e^t, t-1 \rangle$, $t \in [0,1]$.

4. Given the vector field $\vec{F}(x,y) = \left\langle \frac{y}{x} + \sin y, \ln x + x\cos y - 4y \right\rangle$
   a. Show that $\vec{F}$ is conservative.
   b. Find all potential functions of $\vec{F}$.
   c. Evaluate $\int_C \vec{F} \cdot d\vec{R}$, where C is any path from the point $(1,\pi)$ to $(e,0)$.

5. Use Green's Theorem to evaluate the line integral $\oint_C (x^2 + y^2)dx + (x^2 y)dy$, where C is the closed curve determined by $x = y^2$ and $y = -x$.

6. Evaluate the surface integral $\iint_S (2x - 3y + z)dS$, where S is the surface given by the vector equation $\vec{R}(s,t) = \langle 2t, t-s, s \rangle$, where $0 \le s \le 1$ and $0 \le t \le 1-s$.

7. Find the flux of $\vec{F}(x,y,z) = \langle -1, 0, 1 \rangle$ across the portion of the cone $z = \sqrt{x^2 + y^2}$ which is inside the cylinder $x^2 + y^2 = 1$ with upward orientation.

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