M55-LE3-002

Mathematics 55 | Elementary Analysis III
Third Long Exam | Second Semester, AY 2013 -2014

1. Determine whether the given sequence or series is convergent or divergent. *15 points*

   (a) $\left\{\ln\left(\frac{n}{2n+1}\right)\right\}_{n=1}^{\infty}$

   (b) $\sum_{n=1}^{\infty} \ln\left(\frac{n}{2n+1}\right)$

   (c) $\sum_{n=1}^{\infty} 2ne^{-n^2}$

   (d) $\sum_{n=1}^{\infty} \frac{4n-1}{n^3+2n^2}$

   (e) $\sum_{n=0}^{\infty} (-1)^n \frac{2^n}{n+5^n}$

2. *7 points*

   (a) Express $\frac{1}{1-2x}$ as a power series about $x = 0$ and state the largest domain D for which this is valid.

   (b) By integrating the series obtained in (a), show that $\ln(1-2x) = -\sum_{n=0}^{\infty} \frac{2^{n+1}x^{n+1}}{n+1} \forall x \in D$.

   (c) Hence, by substituting an appropriate value of x, deduce the value of $\sum_{n=0}^{\infty} \frac{1}{(n+1)2^{n+1}}$.

3. Let $f(x) = \sum_{n=1}^{\infty} \frac{(x-2)^n}{7^n \sqrt[3]{n}}$ *7 points*

   (a) Find the interval of convergence of f.

   (b) Express $f'(0)$ as a series.

4. Find the third degree Taylor polynomial of $f(x) = \ln x$ about $x = 1$. *4 points*

5. For all real number x, it is well known that $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$ *7 points*

   (a) Obtain a power series for $f(x) = e^{-x^2}$.

   (b) Using the coefficient of x^4 in the series found in (a), deduce the value of $f^{(4)}(0)$.

   (c) Use the first four nonzero terms of the series obtained from (a) to approximate $\int_0^1 e^{-x^2}\,dx$.

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