M55-LE3-003

Mathematics 55 — Elementary Analysis III

Third Long Exam — First Semester, AY 2014 -2015

I. Determine whether each of the following series is convergent or divergent

1. $\sum_{n=1}^{\infty} e^{\frac{1}{n}}$ (2 pts.)
2. $\sum_{n=1}^{\infty} \left(\frac{3+2n^2}{3n^2-2}\right)^n$ (3 pts.)
3. $\sum_{n=2}^{\infty} \frac{1}{n\sqrt[3]{\ln n}}$ (4 pts.)
4. $\sum_{n=1}^{\infty} \frac{2^n}{\cos^2 n + 5^n}$ (4 pts.)

II. Find the sum $\sum_{n=1}^{\infty} \left(\frac{1}{2n} - \frac{1}{2n+2}\right)$. (3 pts.)

III. Find a formula for the general term a_n of the sequence $\left\{\frac{2!}{0!}, \frac{3!}{1!}, \frac{4!}{2!}, \frac{5!}{3!}, \cdots\right\}$, assuming that the pattern of the first few terms continues. Determine whether the sequence converges or diverges. If it converges, find the limit. (3 pts.)

IV. Consider the power series $\sum_{n=1}^{\infty} \frac{n(x+3)^n}{2^n(n^2+1)}$.

1. Show that the radius of convergence is 2. (3 pts.)
2. Find the interval of convergence. (4 pts.)

V. Find the Taylor series of $f(x) = 3^x$ at $a = 2$. Write the answer using summation notation. (4 pts.)

VI. For $|x| < 1, \sum_{n=0}^{\infty} x^n = \frac{1}{1-x}$. (5 pts.)

1. Use the given to show that $\frac{1}{1+x^3} = \sum_{n=0}^{\infty} (-1)^n x^{3n}$, $|x| < 1$.
2. Use the power series in (1) to find a power series representation for $\frac{x^3}{(1+x^3)^2}$, if $|x| < 1$.

VII. Write **TRUE** if the statement is correct and **FALSE** if otherwise. (1 pt. each)

1. Every convergent sequence is monotonic.
2. If the sequence $\{a_n\}$ is decreasing and $a_n > 0$ for all $n \in \mathbb{N}$, then the sequence converges.
3. The series $\sum_{n=1}^{\infty} a_n$ is convergent whenever the series $\sum_{n=1}^{\infty} |a_n|$ is convergent.
4. Given the series $\sum_{n=1}^{\infty} a_n$ of positive terms, if $\lim_{n\to\infty} \frac{a_n}{\frac{1}{n^3}} = L > 0$, then $\lim_{n\to\infty} a_n = 0$.
5. If $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ are divergent series, then the series $\sum_{n=1}^{\infty} (a_n + b_n)$ is divergent.

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