



1. (a) Show that the sequence  $\left\{ \frac{2n^3 - 3}{5n^3 - 2n} \right\}_{n=1}^{\infty}$  is convergent.

(b) Explain why the series  $\sum_{n=1}^{\infty} \frac{2n^3 - 3}{5n^3 - 2n}$  is divergent.

2. Find the sum of the series  $\sum_{n=2}^{\infty} \left[ \frac{2n-2}{n+1} - \frac{2n}{n+2} \right]$ .

3. Determine whether the given series is convergent or divergent.

(a)  $\sum_{n=1}^{\infty} \frac{\cos^4 n}{n^3 + 5n + 2}$

(c)  $\sum_{n=1}^{\infty} \frac{n!}{3^n}$

(b)  $\sum_{n=1}^{\infty} \frac{1 + 3n^3}{1 + 2n^4}$

(d)  $\sum_{n=1}^{\infty} \frac{\ln n}{n}$

4. Determine the radius and interval of convergence of the power series  $\sum_{n=1}^{\infty} \frac{(x-2)^n}{3^n \sqrt{n+1}}$

5. (a) Find a power series representation for  $\frac{1}{2-x}$

(b) Find a power series representation for  $\frac{1}{(2-x)^2}$ . (Hint:  $\frac{1}{(2-x)^2} = D_x \left[ \frac{1}{2-x} \right]$ ).

(c) Use (b) to find the sum of the series  $\sum_{n=0}^{\infty} \frac{(-1)^{n-1} n}{2^{n+1}}$ .

6. Let  $f(x) = \sqrt{x}$ .

(a) Find the third degree Taylor polynomial of  $f(x)$  about  $x = 1$ ,

(b) Using (a), estimate the numerical value of  $\sqrt{1.01}$ .