Mathematics 53: Optimization Problems

Suggestions on Solving a Problem Involving Absolute Extrema

1. If possible, draw a diagram of the problem corresponding to a general situation. Assign variables to all quantities involved.
2. Identify the quantity, say Q, to be maximized or minimized.
3. Formulate an equation involving Q and the other quantities. If more than one variable is involved, use the conditions in the problem to look for relationships among the variables so that some may be eliminated and thereby expressing Q as a function of a single variable.
4. Determine the domain of Q from the physical restrictions of the problem.
5. Use appropriate theorems involving absolute maxima and minima to solve the problem. Then, answer the question.

Exercises

1. Find two numbers which differ by 64 such that their product is minimum.
2. Find the equation of the tangent line to the curve $y = x^3 - 4x^2 + 5x$ with the least slope.
3. Suppose the height from the ground of an aircraft is given by $s(t) = 60t^2 - 6t^3$ feet where t is the time in minutes after take-off.
   (a) What is the greatest height the aircraft attains during the first ten minutes?
   (b) At what moment will it be rising the fastest?
4. A page of a certain book is to contain 27 square inches of print. If the margin at the top, bottom, and the left side are 2 inches and the margin at the other side is 1 inch, determine the dimensions of the smallest piece of paper that can be used.
5. A farmer wants to fence a rectangular plot of area 1800 square feet. He wants to use additional fencing to build two internal divider fences, both parallel to the same outer boundary sections. What is the minimum total length of fencing that this job will require?
6. A manufacturer wishes to construct a closed box of volume 288 cubic inches where the base is a rectangle having length three times it width. Find the dimensions of the box constructed from the least amount of material.
7. A rectangular open tank of volume 125 cubic meters is to have a square base. The cost per square meter for the bottom is P200 and for the sides is P100. Find the dimensions of the tank for the cost of the material to be least.
8. Find the dimensions of the largest rectangle that can be inscribed in a right triangle with sides of length 6 in., 8 in., and 10 in. if two sides of the rectangle lie on the legs of the triangle.
9. Find the area of the largest rectangle that can be inscribed in an isosceles right triangle with legs of length 6, if one side of the rectangle lies on the hypotenuse.
10. Find the dimensions of the largest rectangle that can be inscribed in a circle of radius 10.
11. Find all points on the graph of $f(x) = 20x^3 - 5x^4 - 3x^5$ where the slope of the tangent line to the graph is of f is maximum.
12. Find the minimum distance of a point on the graph of $f(x) = x^2 + 1$ from the point $(0, 3)$.
13. Find the smallest and largest distance of the point $(3, -4)$ from a point on the circle having equation $x^2 + y^2 = 9$.
14. Find the volume of the largest right circular cylinder that can be inscribed in a sphere of diameter 20 inches.
15. The operating cost of a certain truck is $\left(1 + \dfrac{x}{45}\right)$ pesos per kilometer when the truck travels at an average speed of x km./hr. and the driver is being paid at P80 per hour. If the truck is to be driven along a 300 km. highway where the minimum speed is 40 km./hr and the maximum speed is 80 km./hr, what average speed would cost the company the least amount of money? How about if the total cost is to be maximal?
16. A closed cylindrical can is to have volume 2000π cubic centimeters. Find the height and radius of the can that minimizes the material needed for the can.
17. A cone with an open base is to be made from a circular piece of sheet metal of radius 10 m. by cutting out a sector and welding the cut edges of the remaining piece together. What is the maximum possible volume of the resulting cone?