

Mathematics 53: Related Rates

1. Oil from a ruptured tanker spreads in a circular pattern. If the radius of the circle increases at a constant rate of 2 ft./s, how fast is the enclosed area increasing at the end of 90 seconds?
2. If the diagonal of a cube is decreasing at a rate of 3 inches per minute, how fast is the side of the cube decreasing?
3. A spherical snowball is being made so that its volume is increasing at a rate of 8 cubic meters per minute. Find the rate at which the radius is increasing when the snowball is 4 meters in diameter.
4. A car is traveling along a straight road at a speed of 20 m./s. An observer is located at a point 7 m. off to the side of the road. How fast is the distance between the car and the observer increasing at the moment when the car is 24 m. past the point on the road that is closest to the observer?
5. A cylindrical balloon is being inflated at the rate of 12π cubic feet per minute and such that the height of the balloon is three times the radius at any given time. Find the rate at which the height is changing at the moment when the radius is 1 foot.
6. An inverted conical tank with vertex angle 60° has liquid being poured into it at the rate of 1000 cubic inches per minute. At what rate is the level of the liquid rising when the liquid is 10 inches deep?
7. A triangular trough (a container) is 12 ft. long, 3 ft. wide at the top, and 3 ft. deep. If water is poured into the trough at the rate of 10 cubic feet per minute, how fast is the water surface rising when the depth is 2 ft.?
8. JV, who is 6 ft. tall, is walking away from a street light pole 30 ft. high at a rate of 2 ft./s. How fast is his shadow increasing in length when JV is 24 ft. from the pole? How fast is the tip of his shadow moving? To follow the tip of his shadow, at what angular rate must he lift his line of sight when his shadow is 6 ft. long?
9. A man 6 ft. tall is walking at a constant rate of 2 ft./s. away from a building and towards a light source on the ground that is 100 ft. from the building. How far is the man from the light source at the instant when the length of the man's shadow on the building is increasing at a rate of $\frac{1}{3}$ ft./s.?
10. A horizontal trough is 16 m. long, and each of its ends is an isosceles trapezoid with an altitude of 4 m, a lower base of 4 m, and an upper base of 6 m. Water is being poured into the trough at a rate of $10 \text{ m}^3/\text{min}$. How fast is the water level rising when the water is 2 m deep?
11. A ship is located 4 miles from a straight shore and has a radar transmitter that rotates 32 times per minute. How fast is the radar beam moving along the shoreline when the beam makes an angle of 45° with the shore?

12. An isosceles triangle has two sides of length 4 cm. If the included angle is θ , and θ increases at a rate of 2° per minute, how fast is the area of the triangle changing when θ is 30° ? (Hint: Convert all angle measures to radians first.)
13. The minute hand and the hour hand of a clock are 4 inches and 5 inches long, respectively. How fast is the distance between the hands of the clock changing at 3:00?
14. A bird and a worm are on the ground and are $20\sqrt{3}$ feet apart. Deciding it was not hungry, the bird flies away from the worm on a straight path that makes an angle of $\pi/3$ radians with the ground. If the bird is flying at a constant rate of 18 feet per second, at what rate is the angle of elevation of the bird from the worm changing at the instant when the bird is 30 feet above the ground? Indicate whether the angle is increasing or decreasing at that instant.

Some Formulas from Geometry (and Trigonometry)

1. Area of Plane Figures:
 - (a) Parallelogram: $A = bh$
 - (b) Trapezoid: $A = \frac{1}{2}(b_1 + b_2)h$
2. Volume of Solids:
 - (a) Rectangular Parallelepiped (Prism): $V = lwh$
 - (b) Right Circular Cylinder: $V = \pi r^2 h$
 - (c) Right Circular Cone: $V = \frac{1}{3}\pi r^2 h$
 - (d) Prism: $V = Bh$, where B is the area of the base
 - (e) Pyramid: $V = \frac{1}{3}Bh$, where B is the area of the base
3. Surface Area of Solids:
 - (a) Rectangular Parallelepiped (Prism): $2lw + 2wh + 2lh$
 - (b) Sphere: $4\pi r^2$
 - (c) Right Circular Cylinder: $2\pi r h + 2\pi r^2$
 - (d) Right Circular Cone: $\pi r\sqrt{r^2 + h^2} + \pi r^2$
4. Trigonometric Laws
 - (a) Law of Sines: $\frac{a}{\sin \alpha} = \frac{b}{\sin \beta} = \frac{c}{\sin \gamma}$
 - (b) Law of Cosines: $a^2 = b^2 + c^2 - 2bc \cos \alpha$
 - (c) Area of a triangle: $A = \frac{1}{2}ab \sin \gamma$ (γ is the included angle)