# UP SCHOOL OF STATISTICS STUDENT COUNCIL Education and Research 

w

erho.weebly.com $\square$ erhomyhero@gmail.com $\boldsymbol{f} /$ erhoismyhero

Statistics 101
Reviewer for Final Examination

Elementary Statistics
S101-FE-002
I. If X and Y are independent random variables with $\mathrm{E}(\mathrm{X})=2, \mathrm{E}(\mathrm{Y})=2$, $\operatorname{Var}(\mathrm{X})=3$, and $\operatorname{Var}(\mathrm{Y})=1$, solve for the following ( 5 pts ):
a) $\mathrm{E}(3 \mathrm{X}-5)$
b) $\operatorname{Var}(4 \mathrm{Y}-5)$
c) $\mathrm{E}(\mathrm{XY})$
d) $\operatorname{Var}(23 X-7 Y)$
e) $\mathrm{E}(5 \mathrm{X}+75 \mathrm{Y}+230)$
II. Solve for the following:

1. Classify the following as continuous or discrete: ( 5 pts )
a. A=number of robbery cases each year listed in a station of QCPD
b. $B=$ weight of dog food in kilograms supplied by a certain pet store per month
c. $\mathrm{C}=$ number of failing subjects of a student in a semester
d. $\mathrm{D}=$ duration of travel in minutes from house to school
e. $\mathrm{E}=$ number of business permits issued each month in Makati
2. Consider the random experiment where a fair coin is tossed five times. Let $X$ be the total number of heads per trial. Specify the sample space, and construct $F(x)$ and $f(x)$. ( 5 pts )
3. An overseas shipment of 5 cars contains 2 that have slight paint blemishes. If an agency receives 3 of these cars at random, list the elements of the sample space (use letters B and N for "blemished" and "nonblemished" respectively, and then to each sample point assign a value x of the random variable X representing the number of automobiles purchased by the agency with paint blemishes. (Hint: I am not asking for the probability distribution, what you need to do is simple list down the possible samples, and write the corresponding value of x ) ( 3 pts )
4. Find the expected number of boys on a committee of 3 selected at random from 4 boys and 3 girls. (Hint: To get the expected value, you need to have $f(x)$. You need to write a general formula for the probability of selecting $x$ boys and selecting $y=3-x$ girls (since you are selecting 3 people, if you already selected $x$ boys, the number of girls selected will be 3-x) recall how we solved the probabilities in problem 1 in the probability exercise. Note that your $x$ will take on several values.) ( 5 pts)
5. A softdrink machine is regulated so that it discharges an average of 250 ml per cup. If the amount of drink is normally distributed with a standard deviation equal to 15 ml , what is the probability a cup will overflow if 280 ml cups are used?
6. In how many ways can 8 people be seated at a round table if a) They can sit anywhere, b) no particular people must not sit next to each other?
7. Five cards are drawn from a pack of 52 well-shuffled cards. Find the probability that: a) at least two jacks are obtained; b) at least one king is obtained; c) 3 are aces
8. Suppose a random sample of size $n$ will be selected from a large population with mean $\mu$ and standard deviation $\sigma=8$. The researchers wish that there is a .95 chance of selecting a sample whose absolute difference between its sample mean and
$\mu,|\overline{X-}-\mu|$, will be less than 2.5 . What sample size must they choose?
9. An electrical firm manufactures light bulbs that have a length of life that is approximately normally distributed, with mean equal to 800 hours and a standard deviation of 40 hours. Find the Probability that a random sample of 16 bulbs will have an average life of less than 775 hours. (Hint: Use the CLT) (5)
10. Consider a small organization which consists of 5 qualified voters: A1, A2, A3, A4, A5. There are 2 candidates vying for the same position (candidate $\mathrm{W}, \mathrm{Z}$ ). A1, A3, and A5 have already decided to vote for W. Suppose we select a sample of size 3 using SRSWOR. Construct the sampling distribution of $\bar{X}$, where $X_{i}=\left\{\begin{array}{l}1 \text { if } i^{\text {th }} \text { voter in the samplevotes for } Z \\ 0 \text { if } i^{\text {th }} \text { voter in the samplevotes for } W\end{array}\right.$
11. Use the sampling distribution in the previous item to compute for:
a) mean of $\bar{X}$
b) s.e. of $\bar{X}$
c) $\mathrm{P}\left(\left|\bar{X}_{-\mu}\right| \leq 3 / 10\right)$
12. A pharmaceutical company knows that $25 \%$ of the pills they manufacture has an ingredient that is below the minimum strength and thus unacceptable. Suppose a sample of 300 pills is selected using SRSWOR. Find the probabilities of the following:
a. Number of unacceptable pills is equal to the expected number of unacceptable pills.
b. Fewer than 30 will be unacceptable.
c. 50 to 60 pills will be unacceptable.
13. True or False: Write TRUE if the statement is always true, otherwise write FALSE. ( 20 pts )
a. Narrower intervals are associated with more precise estimates.
b. An unbiased estimator is always the best estimator of a parameter.
c. A $95 \%$ confidence interval means that the true parameter has a $95 \%$ chance of being in the interval.
d. A Chi-square test for independence can be used if more than $20 \%$ of the expected frequencies are less than 5 or when any expected frequency is less than one.
e. The Chi-square test for independence is sensitive to sample size.
f. A simple linear regression model relates a response variable $Y$ to an explanatory variable $X$ through a function in any form.
g. The simple linear regression model is a deterministic model.
h. If the p-value is less than the level of significance, then we reject the alternative hypothesis.
i. If we reject Ho at .001 level, then we also reject Ho at .01 level.
j. A decrease in the probability of Type I error one will usually result in a decrease in the probability of the Type II error.
14. Identification ( 5 pts )
a. This is the operational statement in hypothesis testing.
b. This is the error committed when we decide not to reject the null hypothesis when in reality the null hypothesis is false
c. This is the error committed when we decide to reject the null hypothesis when in reality the null hypothesis is true.
d. What is the goal in hypothesis testing?
e. If a $(1-\alpha) 100 \%$ CI for $\mu_{\mathrm{x}}-\mu_{\mathrm{y}}$ is given by $(\mathrm{a}, \mathrm{b})$, what is the $(1-\alpha) 100 \% \mathrm{CI}$ for $\mu_{\mathrm{y}}-\mu_{\mathrm{x}}$ ?
15. Problem Solving || Show complete solutions, provide interpretations for all answers. Follow these steps in hypothesis testing :
i. Ho:
ii. Ha:
iii. $\alpha=$
iv. Test statistic:
v. Computations
vi. Decision Rule:
vii. Conclusion
a. Consider a small groups of 6 people with ages $22,24,31,32,39$, and 54 . The population mean is 32.5 years and the population variance is 64.25 .Suppose a sample of size 3 is selected using SRSWOR. Construct the sampling distribution of the minimum. Is the minimum an unbiased estimator for $\mu$ in this case?
b. A medical researcher wishes to determine if a pill has the undesirable side effect of reducing the blood pressure of the user. The study involves recording the initial blood pressures of 15 college-age women. After they use the pill regularly for six months, their blood pressures are again recorded. The researcher wishes to draw inferences about the effect of the pill on blood pressure from the observations given the table below:

|  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| Before <br> $(\mathrm{x})$ | 70 | 80 | 72 | 76 | 76 | 76 | 72 | 78 | 82 | 64 | 74 | 92 | 74 | 68 | 84 |
| After (y) | 68 | 72 | 62 | 70 | 58 | 66 | 68 | 52 | 64 | 72 | 74 | 60 | 74 | 72 | 74 |

i. Calculate a $95 \% \mathrm{CI}$ for the mean reduction in blood pressure.
c. A course in mathematics is taught to 12 students by the conventional classroom procedure/ A second group of 10 students was given the same course by means of programmed materials. At the end of the semester, the same exam was given to each group. The 12 students meeting in the classroom made an average grade of 85 with a standard deviation of 4 , while the 10 students using programmed materials made an average grade of 81 with a standard deviation 5. Test the hypothesis that the two methods of learning are equal using a .10 level of significance. Assume that the populations are normally distributed with equal variances.
d. A manufacturer of sports equipment has developed a new synthetic fishing line that he claims has a mean breaking strength of 8 kilograms with a standard deviation of .5 kilogram. A random sample of 50 lines is tested and found to have a mean breaking strength of 7.8 kilograms. Will this sample provide enough evidence to reject the manufacturer's claim at .01 level of significance?
e. Suppose a store manager wishes to be $95 \%$ confident that his estimate for the mean monthly family grocery expense is correct to within $\pm 700$ pesos. Based on prior information, the store manager believes that the monthly family grocery expenses follow a normal distribution, and he has arrived at an estimate of 7,000 for the standard deviation of the population. Determine the sample size needed to meet his criteria.
f. In an experiment to study the dependence of hypertension on smoking habits, the following data were taken on 200 individuals:

|  | Nonsmokers | Moderate <br> Smokers | Heavy <br> Smoker |
| :--- | :--- | :--- | :--- |
| With <br> Hypertension | 21 | 36 | 30 |
| No <br> Hypertension | 48 | 36 | 29 |

Test the hypothesis that the presence or absence of hypertension is independent of smoking habits.

