S115_LE1_001

Statistics 115 — January 14, 2013
First Long Examination — TGCapistrano

I. TRUE OR FALSE. Write 'TRUE' if the statement is always true; otherwise, write 'FALSE'.

1. If a die is tossed twice then P(A)=1 where A is the event that the sum of the number of dots on the face that comes up on the first toss with the number of dots on the second toss is at least 2.
2. If a die is tossed twice then P(B)=1/36 where B is the event that the sum of the number of dots on the face that comes up on the first toss with the number of dots on the second toss is exactly 2.
3. If A_1 is the event that the selected student is in Grade 1 and A_2 is the event that the selected student is in Grade 2 then $P(A_1 \cup A_2)=P(A_1)+P(A_2)$.
4. If the sample space is an infinite collection then it is not a discrete sample space.
5. If f( ) is the probability mass function of the discrete random variable X then f(x)>0 for any real number x that belongs in the range of X.
6. If f( ) is the probability density function of the continuous random variable X then f(x)=P(X=x) for any real number x.
7. According to the Central Limit Theorem, the sampling distribution of the sample mean will be approximately normal when n is sufficiently large so long as the parent population from where the random sample of size n came from is normally distributed.
8. If $(X_1, X_2, \ldots, X_n)$ is a random sample from a normal distribution with population mean and variance equal to 1 and 4, respectively, then the mean of the sample mean, $\bar{X}$, is equal to 1.
9. When a sample of size n is selected using simple random sampling without replacement (SRSWOR) and X_i is the measure taken from the i^{th} selected element in the sample then $(X_1, X_2, \ldots, X_n)$ is a random sample from a finite population.
10. When a sample of size n is selected using simple random sampling with replacement (SRSWR) and X_i is the measure taken from the i^{th} selected element in the sample then $(X_1, X_2, \ldots, X_n)$ is a random sample from a finite population.
11. If X_1~Normal($\mu=2,\sigma^2=4$), X_2~Normal($\mu=2,\sigma^2=16$), X_3~Normal($\mu=2,\sigma^2=25$) and X_1, X_2, X_3 are independent random variables then (X_1, X_2, X_3) is a random sample from an infinite population.
12. If the standard error of $\bar{X}$ is close to 0 then this indicates that the values of $\bar{X}$ from one sample to the other do not vary much from each other.
13. In SRSWOR, the finite population correction (fpc) that appears in the formula of the standard error of $\bar{X}$ will approach 0 as the size of the population approaches infinity while the size of the sample is fixed.
14. The standard error of the sample mean, $\bar{X}$, is smaller if a sample of size 25 is selected using SRSWOR instead of SRSWR.
15. If X~Normal($\mu=2,\sigma^2=4$), then P(X=2)=0.

II. FILL IN THE BLANKS.

1. The probability distribution of a statistic is called its __________________.
2. If $(X_1, X_2, \ldots, X_{400})$ is a random sample from an infinite population whose mean $\mu=16$ and variance $\sigma^2=196$ then the value of the standard error of the sample mean $\bar{X} = \sum_{i=1}^{400} X_i / 400$ is __________.
3. If $f_n(A)$ is the relative frequency of occurrence of event A when the random experiment is repeated n times under uniform conditions then the P(A)=_________ by the *a posteriori* definition of the probability of event A.
4. If we select a sample of size 4 from a population containing 24 elements using SRSWOR, then the number of elements in the sample space that contains all possible samples denoted by sets containing 4 distinct elements is _____________.
5. If we select a sample of size 4 from a population containing 24 elements using systematic sampling, then the number of elements in the sample space that contains all possible samples denoted by sets containing 4 distinct elements is ___________.
6. If X is a random variable and F(·) is its cumulative distribution function of X (CDF) then for any real number x, F(x) = _____________.
7. If Z is a standard normal random variable then $P(Z < -z_\alpha)$ = ___________.
8. If X~t(v) then $P(-t_\alpha(v) < X < t_\alpha(v))$ = _________.
9. The value of $t_{0.90}(v=10)$ is _________.
10. If X~t(v) then the limiting distribution of X is _______________ as v approaches infinity.

III. PROBLEM SOLVING. Show all the important steps. Always show the formulas used in answering the following problems with the appropriate values plugged-in.

1. Suppose X is a continuous random variable and its CDF is:

$$F(x) = \begin{cases} 0 & \text{when } x \le 0 \\ \dfrac{x}{2} & \text{when } 0 < x \le 1 \\ 1 - \dfrac{1}{2x} & \text{when } x > 1 \end{cases}$$

Compute for the following probabilities:

a) $P(X > 1.6)$
b) $P(X < 0.25)$
c) $P(X > -2)$
d) $P(0.5 < X < 2)$
e) $P(0.1 \le X \le 0.6)$

2. Let X=height of the selected adult in the population. Suppose it is known that X is normally distributed with mean of 174.5 cm and standard deviation of 8 cm.

a) Compute for the proportion of adults in this population who are taller than 196.5 cm.
b) Compute for the probability of selecting an adult whose height is at least 197.3 cm.

3. There are 7 elements in the population and each one is assigned a serial number from 1 to 7. The elements labeled 1, 2, 3 are all from Luzon, while the elements labeled 4 and 5 are from the Visayas, and the elements labeled 6 and 7 are from Mindanao. Suppose a sample of size 5 is selected from this population using SRSWOR.

a) Specify the sample space by roster method using the notation, $\{x_1, x_2, x_3, x_4, x_5\}$ where the x_is are the serial numbers of the selected elements, to denote a sample of size 5.
b) Let A= event of selecting a sample where none of the elements are from Mindanao Specify event A by roster method.
c) Using your answer in (b), compute for P(A). Present final answer as a fraction.
d) Define X=number of elements in the sample who are from Mindanao. Specify the event, X=2, by roster method.
e) Using your answer in (d), compute for P(X=2). Present final answer as a fraction.

4. Define X_i = tensile strength of the i^{th} selected metal component manufactured by Metallica Corp (in kg/cm^2), i=1,2,...,25. Suppose $X_i \sim \text{Normal}(\mu=10{,}000,\ \sigma^2=100^2)$ and $(X_1,X_2,\ldots,X_{25})$ is a random sample.

a) Compute for the probability of the event that the tensile strength of the first selected metal component (X_1) is at least 10,125 kg/cm^2.
b) Compute for $P(9{,}975 < \bar{X} < 10{,}050)$.
c) Compute $P(|\bar{X} - \mu| \le 50)$.
d) Compute for the probability of selecting a sample whose sample variance $S^2 = \sum_{i=1}^{25}(X_i - \bar{X})^2 / 24$ is greater than 5,770 (kg/cm^2)^2.
e) Suppose the population variance is unknown. Compute for the probability of selecting a sample whose sample mean, $\bar{X} = \sum_{i=1}^{25} X_i / 25$, is less than 9,967.05 kg/cm^2 if the sample standard deviation of the selected sample is equal to 125 kg/cm^2.