S115_LE2_001

Statistics 115 | February 18, 2013
Second Long Examination | TGCapistrano

I. TRUE OR FALSE. Write 'TRUE' if the statement is always true; otherwise, write 'FALSE'.

1. A parameter can have more than one unbiased estimator.
2. If T is an unbiased estimator of μ then we are assured that the computed estimate using T is close to μ.
3. If T is a reliable estimator of μ then we are assured that the computed estimate using T is close to μ.
4. The standard error of the statistic T is a measure of reliability of T.
5. The sample standard deviation is an unbiased estimator of the population standard deviation when the sample is selected using simple random sampling with replacement.
6. The sample mean is an unbiased estimator of μ under simple random sampling with replacement and under simple random sampling without replacement.
7. The sample mean is the most efficient estimator of μ when sampling from the normal distribution.
8. When the margin of error of the sample proportion is equal to 0.03 then if we repeatedly take samples of size n from the same population and each time measure the error of estimation of P using the sample proportion, then the computed error of estimation will exceed 0.03 α100% of the time.
9. The 95% confidence interval estimate for μ based on a random sample of size 50 from a normal distribution with known variance is always longer than the 95% confidence interval estimate based on a random sample of size 100 from the same population.
10. The 95% confidence interval estimate for μ based on a random sample of size 50 from a normal distribution with known variance is always longer than the 90% confidence interval estimate based on a random sample of size 50 from the same population.
11. If (T_1, T_2) is a 95% confidence interval estimator for μ then $P(T_1 < \mu < T_2)=0.95$.
12. If we repeatedly take samples of size n from the same population and each time compute for the interval estimate for μ using a 95% confidence interval estimator for μ then 95% of the computed interval estimates will contain the value of μ.
13. Saying that we are 95% confident that the value of μ is in the interval, (15,20), means that the probability that the value of μ is in the interval, (15,20), is equal to 0.95.
14. The probability that the computed interval estimate contains the value of the parameter of interest can only be determined if we know the value of the parameter.
15. The confidence interval estimator of σ^2 when we sample from a normal distribution is sensitive to the assumption of normality.

II. COMPUTATIONS. Always show the important steps involved in the computations. When asked for the interval estimate, always present the interval estimators and the plugged-in values. No immediate rounding-off. Whenever necessary, round off final answer to 3 decimal places.

1. A television network collected data on the number of hours in the past week individuals spend watching television from a random sample of 50 people. The results appear below:

11	17	33	19	47	56	39	18	26	28
5	26	6	45	9	26	7	16	43	33
38	9	35	27	29	13	15	17	3	41
12	38	36	15	25	32	42	45	41	38
40	25	25	1	41	26	5	57	11	27

Use the sample data to estimate the following parameters:

a) mean number of hours in the past week individuals spend watching television
b) standard error of the point estimator used in (a)
c) proportion of individuals who spend more than 20 hours watching television in the past week
d) standard error of the point estimator used in (c)

2. Carlton Sign Company wanted to know the variance of the life of the light fixtures it uses in its signs. It selected a random sample of 25 signs and learned that the fixtures in the sample lasted an average of 9,500 hours with a standard deviation of 81 hours. Assuming that the distribution of the life of a light fixture is normally distributed, compute for a 90% confidence interval estimate for the population variance.

3. A manufacturer of office machines is considering the production of a new word processor. The decision to start large-scale production of the new machines will be based on the comparison of the mean operating speed using the standard machines (μ_X) and the mean operating speed using the new machines (μ_Y). Since operators of the machine have varying abilities, a random sample of 20 typists was selected and the speed of each typist in the sample was observed once using the new word processor and once using the standard word processor. The collected data on the speed (in minutes) are as follows:

Typist (i)	1	2	3	4	5	6	7	8	9	10
Standard Processor (X_i)	60.2	58.7	59.4	60.3	61.7	60.2	64.1	63.2	62.4	57.8
New Processor (Y_i)	57.2	57.4	56.4	58.5	60.1	61.4	61.9	60.4	60.0	56.8

Typist (i)	11	12	13	14	15	16	17	18	19	20
Standard Processor (X_i)	55.4	61.2	64.7	64.1	62.9	65.8	69.3	56.4	58.5	63.7
New Processor (Y_i)	50.2	58.4	63.5	60.5	62.2	66.3	68.5	56.6	58.3	60.2

a) Assuming normality, compute a 95% confidence interval estimate for $\mu_X - \mu_Y$.

b) Using the result in (a), does the manufacturer have sufficient evidence that the mean operating speed using the standard machines is different from the mean operating speed using the new machines?

4. A firm is choosing between two machines to fill containers with a chemical solution. Since the solution is caustic, the firm wishes to buy the machine that produce the least waste. Thus, the firm wishes to compare the proportion of containers that are overfilled using Machine 1 (p_1) with the proportion of containers that are overfilled using Machine 2 (p_2). They collected two independent random samples from the two populations and the results are as follows:

Machine	Number Overfilled	Sample Size
1	60	250
2	88	275

a) Compute an approximate 95% confidence interval estimate for $p_1 - p_2$.

b) Using the result in (a), does the firm have sufficient evidence that the proportion of containers that are overfilled using Machine 1 is different from the proportion of containers that are overfilled using Machine 2?

5. A study is conducted between high school students and college students to compare their proficiency at writing computer programs for microcomputers. For this study, the researchers wish to compare the mean time (in minutes) of high school students to write an error-free program (μ_X) with the mean time (in minutes) of college students to write an error-free program (μ_Y). Data taken from two independent samples were summarized as follows:

Statistics	High school	College
Mean time	70	84
Standard deviation	10	12
Sample size	10	10

Assuming normality of both populations, compute a 90% confidence interval estimate for $\mu_X - \mu_Y$. Use the appropriate confidence interval estimator considering that the two sample sizes are equal to each other.

Formulas:

$$S_p^2 = \frac{(n_1-1)S_X^2 + (n_2-1)S_Y^2}{n_1+n_2-2}$$

$$v = \frac{\left(\frac{s_X^2}{n_1} + \frac{s_Y^2}{n_2}\right)^2}{\frac{\left(\frac{s_X^2}{n_1}\right)^2}{n_1-1} + \frac{\left(\frac{s_Y^2}{n_2}\right)^2}{n_2-1}}$$