S115_LE3_001

Statistics 115 March 18, 2013
Third Long Examination TGCapistrano

I. TRUE OR FALSE. Write 'TRUE' if the statement is always true; otherwise, write 'FALSE'.

1. The probability of committing a Type I error when we decide to reject the null hypothesis will never be larger than the level of significance.
2. The probability of committing a Type II error when we decide to reject the null hypothesis will always be 0.
3. If A=acceptance region at 0.05 level of significance and B=acceptance region of the same test at 0.01 level of significance then $A \subset B$.
4. If Ho is not rejected at 0.05 level of significance then Ho will not be rejected at 0.01 level of significance using the same test on the same data set.
5. If Ho is rejected at 0.05 level of significance then Ho will also be rejected at 0.01 level of significance using the same test on the same data set.
6. If Ho is rejected at 0.01 level of significance then Ho will also be rejected at 0.05 level of significance using the same test on the same data set.
7. In testing Ho: $\mu=\mu_o$ against Ha: $\mu<\mu_o$ at α level of significance based on a random sample from a normal distribution whose σ is known, the probability of committing a Type II error decreases as sample size n increases.
8. In testing Ho: $\mu=\mu_o$ against Ha: $\mu<\mu_o$ at α level of significance based on a random sample from a normal distribution whose σ is known, the probability of committing a Type II error decreases as the true value of μ approaches μ_o.
9. In testing Ho: $\mu=\mu_o$ against Ha: $\mu<\mu_o$ at α level of significance based on a random sample from a normal distribution whose σ is known, the p-value becomes smaller as the computed value of the test statistic becomes larger.
10. In testing Ho: $\mu=\mu_o$ against Ha: $\mu>\mu_o$ at α level of significance based on a random sample from a normal distribution whose σ is known, the p-value becomes smaller as the computed value of the test statistic becomes larger.
11. If the value of the test statistic of the 2-tailed test, Ho: $\mu=0$ vs Ha: $\mu\neq0$, belongs in the acceptance region at 0.01 level of significance then the computed 99% confidence interval estimate for μ using the same sample data will contain 0.
12. If we decided to reject Ho: $\mu=0$ against the alternative Ha: $\mu\neq0$ but the true value of μ is 1 then we committed a Type I error.
13. If we test Ho: $\mu=0$ vs Ha: $\mu\neq0$ and the value of the test statistic does not belong in the critical region at .05 level of significance then we decide to accept Ho and conclude that the data provide us with sufficient evidence to conclude that $\mu=0$ at .05 level of significance.
14. If the computed value of the test statistic is 2.5 to test Ho: $\mu=0$ vs Ha: $\mu\neq0$ at α level of significance based on a random sample of size 10 from a normal distribution with unknown variance σ^2, then the p-value is just the same as the p-value if the computed value of the test statistic is -2.5.
15. The chi-squared test for the population variance is sensitive to the assumption of normality.

II. COMPUTATIONS. Show important parts of your solution. No immediate rounding-off. Whenever necessary, round-off final answer only to four decimal places.

1. It is claimed that an automobile is driven on the average less than 20,000 kilometers per year. To test this claim, a random sample of 18 automobile owners was selected and the owners were asked to keep a record of the kilometers they travel.
   a) State Ho and Ha. Define the parameters.
   b) Suppose that the sample comes from a normal distribution but the value of the population standard deviation is unknown.
      i. What is the null distribution of the test statistic? Indicate the values of the parameters of this distribution.
      ii. Determine the p-value if the computed value of the test statistic is -2.11.
      iii. Based on this p-value, should Ho be rejected at 0.02 level of significance?
   c) Suppose that the sample comes from a normal distribution and this time the value of the population standard deviation is known to be equal to 3,900 kilometers.
      i. What is the null distribution of the test statistic? Indicate the values of the parameters of this distribution.
      ii. Determine the p-value if the computed value of the test statistic is -2.11.
      iii. Based on the p-value in (ii), should Ho be rejected at 0.02 level of significance?
      iv. Compute for the p-value if the computed value of the test statistic this time is 2.11.

2. A research organization selected two independent samples from two age groups to test whether a new television show has greater appeal to older people than younger people. The selected elements in both samples were asked to attend a preview performance of the show at the television studio and were asked if they enjoyed the show or not. The research organization summarized the collected data in the following frequency distribution table:

	Age groups	
Response	Under 35	35 & over
Enjoyed the show	47	180
Did not enjoy the show	53	120

   a) State Ho and Ha. Define the parameters.
   b) Write the formula of the test statistic to be used.
   c) State the decision rule at 0.05 level of significance
   d) Compute for the value of the test statistic.
   e) Is there sufficient evidence at 0.05 level of significance for the research organization to conclude that the proportion among the older age group who enjoyed the show is larger than the proportion among the younger age group who enjoyed the show?

3. It is known that the mean height of all the males in the freshman class of a certain university 25 years ago is 165.5 centimeters. A random sample of 20 males in the present freshman class was selected to determine if the males in the present freshman class are generally taller than the males in the freshman class 25 years ago. The following data are the heights of the males in the sample:

170.2	168.4	150.5	172.3	154.5	185.2	175.1	165.1	166.8	154.2
165.4	168.5	170.3	166.4	167.8	160.0	170.5	160.4	168.2	160.4

a) State Ho and Ha. Define the parameters.
b) Write the formula of the test statistic to be used assuming normality.
c) State the decision rule at 0.05 level of significance
d) Compute for the value of the test statistic.
e) Is there sufficient evidence at 0.05 level of significance supporting the hypothesis that the males in the present freshman class are generally taller than the males in the freshman class 25 years ago?

4. The machine used to fill up the containers of a particular lubricant is set so that the mean amount dispensed is 10 liters with a standard deviation of 0.15 liters. A random sample of 10 containers was selected because it is suspected that there has been a change in the standard deviation of the amount dispensed which means they would have to recalibrate the machine. The values below are the contents (in liters) of the 10 containers in the sample:

10.2	9.7	10.1	10.3	10.1
9.8	9.9	10.4	10.3	9.8

a) State Ho and Ha. Define the parameters.
b) Write the formula of the test statistic to be used assuming normality.
c) State the decision rule at 0.01 level of significance
d) Compute for the value of the test statistic.
e) Is there sufficient evidence at 0.01 level of significance supporting the suspicion that the standard deviation is not anymore 0.15 liters?

5. A heavy machinery manufacturer produces two metal-stamping machines. Since both machines perform the same functions, the manufacturer wishes to continue producing only one of them. The two machines are to be compared by measuring their amount of downtime (time not in use because of mechanical failure) each week. A random sample of 12 machine operators was chosen and each operator is assigned to Machine A for one week and Machine B for another week (order of assignment is determined by a randomization mechanism). The manufacturer will stop producing Machine A if there is sufficient evidence at 0.01 level of significance that the mean downtime for Machine B is less than the mean downtime for Machine A. The table below shows the recorded amount of downtime (in minutes) of both machines for each operator in the sample:

Operator	1	2	3	4	5	6	7	8	9	10	11	12
Machine A	27	30	28	25	35	26	20	15	22	38	25	36
Machine B	25	31	24	26	37	24	15	14	19	26	20	30

a) State Ho and Ha. Define the parameters.
b) Write the formula of the test statistic to be used assuming normality.
c) State the decision rule at 0.01 level of significance
d) Compute for the value of the test statistic.
e) Is there sufficient evidence at 0.01 level of significance supporting the belief that the mean downtime for Machine B is smaller than the mean downtime for Machine A?

6. The braking ability was compared for two different models of cars. Two independent random samples of size 12 were selected for each model and the cars in the two samples were tested. The recorded measurement was the distance required to stop when the brakes were applied at 40 miles per hour. The data will be used to determine if there is a difference in the mean stopping distance for the two models of cars. The table below shows the computed summary measures for both samples:

Statistics	Model	
	A	B
Mean	118	109
Standard deviation	10 2	8 7
Sample size	12	12

a) State Ho and Ha. Define the parameters.
b) Write the formula of the test statistic to be used assuming normality.
c) State the decision rule at 0.05 level of significance
d) Compute for the value of the test statistic.
e) Is there sufficient evidence at 0.05 level of significance supporting the belief that mean stopping distance for the two models of cars are different?

Formulas:

$$S_P^2 = \frac{(n_1 - 1)S_X^2 + (n_2 - 1)S_Y^2}{n_1 + n_2 - 2}$$

$$v = \frac{\left(s_X^2 / n_1 + s_Y^2 / n_2 \right)^2}{\frac{\left(s_X^2 / n_1 \right)^2}{n_1 - 1} + \frac{\left(s_Y^2 / n_2 \right)^2}{n_2 - 1}}$$