S117_FIN_001

Statistics 117 Mathematics for Statistics
Final Long Examination TGCapistrano

I. FILL IN THE BLANKS

1. The minimal sigma-field containing $\mathscr{C}$ denoted by $\sigma(\mathscr{C})$, is defined as a collection of subsets of Ω that satisfies the following:
   a) __________
   b) __________
   c) __________

2. The field, denoted by $\mathscr{F}$, is defined as a collection of subsets of Ω that satisfies the following:
   a) __________
   b) If ______ then _______
   c) If ______ then _______

3. The class $\{A_1, A_2, ..., A_n\}$ where $A_i \subset \mathscr{A} (i = 1, 2, ..., n)$ is defined to be a partition of A if and only if it satisfies the following:
   a) __________
   b) __________

4. The limit of a monotone nondecreasing sequence of sets $\{A_n\}$ is __________.
5. The limit of a monotone nonincreasing sequence of sets $\{A_n\}$ is __________.

6. The steps in proving the statement P(n) is true for all positive integers n by mathematical induction are:
   a) Step 1: __________
   b) Step 2: __________
   c) Step 3: __________
   Step 4: Conclude that P(n) is true for all positive integer n

7. In proving the statement p→q
   a) By direct method, we assume that _________ is/are true
   b) By contrapositive, we assume that _________ is/are true
   c) By contradiction, we assume that _________ is/are true

8. In disproving p→q, we need to show a counterexample for which _________ is true

9. In proving $A \subset B$, we need to show that $\omega \in A$ implies _________

10. In proving that the sets A, B and C are pairwise disjoint, we need to show
    a) __________
    b) __________
    c) __________

11. Suppose an urn contains M balls of which K are defective and (M-K) are nondefective
    a) The number of ordered samples with replacement of size n containing r defectives and (n-r) nondefectives _________.
    b) The number of ordered samples without replacement of size n containing r defectives and (n-r) nondefectives _________.
    c) The number of unordered samples without replacement of size n containing r defectives and (n-r) nondefectives _________.

12. a) $\sum_{x=0}^{n} \binom{n}{x} a^{x-1} =$ ______
    b) $\sum_{x=0}^{\infty} \frac{a^0}{x!} =$ ______
    c) $\sum_{x=0}^{\infty} \binom{n+x-1}{x} a^x =$ ______ when -1< a < 1
    d) $\sum_{x=1}^{n} 3x =$ ______
    e) $\sum_{x=100}^{150} y =$ ______
    f) $\sum_{x=1}^{\infty} \frac{5}{x(x+1)} =$ ______
    g) $\sum_{r=1}^{n} ar^1 =$ ______ when r ≠ 1

13. Let Ω = set of real numbers. Let $f(x) = (2x)|_{[-1,1]}(x) + (x-3)|_{[0,\infty]}(x)$.
    a) f(1) = _________
    b) f(-3) = _________

14. The number of ways of forming 5 committees with 3 members each is _________.
15. The number of distinct permutations of the word 'character' is __________.
16. The number of distinct outcomes when we toss a die 10 times is __________.
17. The number of 5-digit numbers that can be formed using the integers 0 to 9 is ___________.
18. An urn contains 10 balls marked from 1-10. Two balls will be selected using random sampling with replacement.
    Let $\Omega = \left\{ \left(\omega_1, \omega_2 \right) : \omega | \in \{1, 2, ..., 10\} \; i = 1, 2 \right\}$
    A_{ij} = the set containing outcomes where ball #i is selected on the jth draw, i=1,2,…,10 and j=1,2.
    a) The elements of the set A_{11} are ________.
    b) The elements of the set $A_{11^c} \cap A_{12}$ are ________.
    c) The set containing outcomes where ball #1 is not selected in any of the 2 draws can be represented using the A_{ij}'s as ________.

II. TRUE OR FALSE. Write 'True' if the answer is always true; otherwise, write 'False'.

1. The set of real numbers is equivalent to the set of counting numbers.
2. The set of positive integers is equivalent to the set of all integers.
3. The set of rational numbers is a non-denumerable set.
4. The set of all outcomes of tossing a die 1,000,000 times is a finite set.
5. f(x) = $x^2 - 1$ is a one-to-one but onto function from the set of positive real numbers into the set of all real numbers.
6. f(x) = $|x-1|$ is a onto but not one-to-one function from the set of nonnegative integers into the set of all nonnegative integers.
7. f(x) = log(x) is a one-to-one but onto function from the set of positive real numbers into the set of all real numbers.
8. Any subset of the set of all real numbers is a Borel set.
9. The Borel field is the minimal σ-field containing the class $\mathscr{C}$ = {(x,y): where x and y are real numbers and x < y)}.
10. A field is closed under finite intersection.
11. A field is closed under countable intersection.
12. The Borel field is closed under finite union.
13. The Borel field is closed under countable intersection.
14. A field is a sigma-field.
15. To disprove the statement "$\forall x, P(x)$" is true, it is enough to provide an example for which the proposition P(x) is false for some x in the domain of discourse.
16. To prove the statement "$\exists x, P(x)$" is true, it is enough to provide an example for which the proposition P(x) is true for some x in the domain of discourse.
17. The proposition $p \vee \sim p$ is a tautology.
18. The proposition $p \vee p$ is a tautology.
19. The propositions $P \equiv (p \equiv q)$ and $Q \equiv (p \wedge q) \vee (\sim p \wedge \sim q)$ are logically equivalent to each other.
20. The propositions $P \equiv ((p \vee q) \wedge r)$ and $Q \equiv (p \vee q)$ are logically equivalent to each other.
21. $(p \vee q) \wedge r) \rightarrow (p \vee q)$
22. $\sim(p \rightarrow q) \leftrightarrow (p \wedge \sim q)$
23. (x=0 and y=0) is a sufficient condition for xy=0.
24. (x=0 and y=0) is a necessary condition for xy=0.
25. If xy≠0 then (x≠0 and y≠0).
26. If xy≠0 then (x≠0 or y≠0).
27. A function f: A→B is called onto iff its range is equal to its codomain.
28. If $A \cap B \cap C = \varnothing$ then the class {A, B and C} is pairwise disjoint.
29. If the class {A,B,C,} is a pairwise disjoint then $A \cap B \cap C = \varnothing$.
30. For any sets A, B and C, $n(A \cup B \cup C) = n(A) + n(B) + n(C)$.
31. If $B \subset A$ then $n(A \cap B^c) = n(A) - n(B)$.
32. If $B \subset A$ then $A^c \subset B^c$.
33. If $(A \cap B) \cup C = C$ then $A \cap B \subset C$.
34. A class that is closed under finite union will also be closed under countable union.
35. A class that is closed under countable union will also be closed under finite union.

36. If $\omega \in \bigcup_{\lambda=\Delta} A_\lambda$ then $\omega \in \bigcap_{\lambda=\Delta} A_\lambda$.
37. $(A-B)^c = A^c - B^c$.
38. The cardinal of the set of positive integers is the same as the cardinal of the set of positive real numbers.
39. The nth term of geometric progression is $a_n = a_1 r^n$ where a_1 is the 1st term of the sequence and r is the common ratio.
40. I think I will get a high grade in this exam.