S117-FE-002

Statistics 117 — Mathematics for Statistics

Final Exam — TGCapistrano

I. FILL IN THE BLANKS.

1. The minimal sigma-field containing C_1 denoted by σ(C), is defined as a collection of subsets of Ω that satisfies the following:
   a)________________________________
   b)________________________________
   c)________________________________
2. The field, denoted by F, is defined as a collection of subsets of Ω that satisfies the following:
   a)________________________________
   b) If ________________________, then ______________________.
   c) If ________________________, then ______________________.
3. The class $\{A_1, A_2, \ldots, A_n\}$ where $A_i \subset A$ (i=1,2,…n) is defined to be a partition of set A if and only if it satisfies the following:
   a)________________________________
   b)________________________________
4. The limit of a monotone nondecreasing sequence of sets $\{A_n\}$ is ___________________.
5. The limit of a monotone nonincreasing sequence of sets $\{A_n\}$ is ___________________.
6. The steps in proving that statement P(n) is true $\forall \mathbb{Z}^{+} n$ by mathematical induction are:
   a) Step 1: _______________
   b) Step 2: _______________
   c) Step 3: _______________
   d) Step 4: Conclude that P(n) is true for all positive integer n.
7. In proving the statement $p \rightarrow q$,
   a) By direct method, we assume that ___________ is/are true.
   b) By contradiction, we assume that ___________ is/are true.
   c) By contrapositive, we assume that ___________ is/are true.
8. In disproving $p \rightarrow q$, we need to show a counterexample for which __________ is/are true.
9. In proving $A \subset B$, we need to show that $\omega \in A$ implies __________.
10. In proving that A, B and C are pairwise disjoint, we need to show:
    a) ________________
    b) ________________
    c) ________________
11. Suppose an urn contains M balls for which K are defective and M-K are non-defective.
    a) The number of ordered samples with replacement of size n containing r defectives and n-r non-defectives is __________.
    b) The number of ordered samples without replacement of size n containing r defectives and n-r non-defectives is __________.
    c) The number of unordered samples without replacement of size n containing r defectives and n-r non-defectives is __________.

12. a) $\sum_{x=0}^{n}\binom{n}{x}a^{x-1} =$ ________.   d) $\sum_{i=0}^{20}\binom{20}{i}\frac{3^{i+1}}{2} =$ ________.

b) $\sum_{j=0}^{\infty}\binom{10+j}{j}0.2^{j} =$ ________.   e) $\sum_{j=1}^{100}(3^{2j}+3j) =$ ________.

c) $\sum_{j=0}^{\infty}\frac{2^{j}}{j!}j =$ ________.   f) $\sum_{x=3}^{\infty}\frac{3}{x^2+x} =$ ________.

13. Let Ω =set of real numbers. Let $f(x) = (2x)I_{[-1,1]}(x) + (x-3)I_{(0,+\infty)}(x)$
    a) $f(1) =$ __________   b) $f(-3) =$__________.
14. The number of ways of forming 5 committees with 3 members each is __________.
15. The number of distinct permutations of the word 'CHARACTER' is __________.
16. The number of distinct outcomes when we toss a die 10 times is __________.
17. The number of 5-digit numbers that can be formed using the integers 0 to 9 is _________.
18. An urn contains 10 balls marked from 1-10. Two balls will be selected using simple random sampling with replacement. Let $\Omega = \{(\omega_1, \omega_2) : \omega_i \in \{1, 2, \ldots 10\}, i = 1, 2\}$. Define A_{ij} = set containing outcomes where the ith ball is selected on the jth draw.
    a) The elements of set A_{11} are __________.
    b) The elements of set $A_{11}^C \cap A_{12}$ are __________.
    c) The set containing outcomes where the 1st ball is not selected on any of the draws can be represented using tha A_{ij}'s as __________.

II. TRUE OR FALSE.

1. The set of real numbers is equivalent to the set of counting numbers.
2. The set of positive integers is equivalent to the set of integers.
3. The set of rational numbers is a non-denumerable set.
4. The set of all outcomes of tossing a coin 1,000,000 times is a finite set.
5. $F(x) = x^2 - 1$ is one-to-one but not an onto function from the set of positive real numbers to the set of all real numbers.
6. $G(x) = |x - 1|$ is onto but not one-to-one function from the set of nonnegative integers to the set of nonnegative integers.
7. If A is a field, then A is a sigma-field.
8. A subset of the set of all real numbers is a Borel set.
9. The Borel field is a minimal field.
10. A field is closed under finite intersection.
11. A field is closed under countable intersection.
12. The Borel field is closed under finite union.
13. The Borel field is closed under countable intersection.
14. A – B and B – A are pairwise disjoint.

15. To disprove the statement $\forall x, P(x)$ is true, it is enough to provide an example for which the proposition P(x) is false for some x in the domain of discourse.
16. To prove the statement $\exists x, P(x)$ is true, it is enough to provide an example for which the proposition P(x) is true for some x in the domain of discourse.
17. The proposition p v ~p is a tautology.
18. The proposition p v p is a tautology.
19. The propositions $P \equiv (p \equiv q)$ and $Q \equiv (p \wedge q) \vee (\sim p \wedge \sim q)$ are logically equivalent to each other.
20. The propositions $P \equiv ((p \vee q) \wedge r)$ and $Q \equiv (p \vee q)$ are logically equivalent to each other.
21. $((p \vee q) \wedge r) \rightarrow (p \vee q)$
22. $\sim (p \rightarrow q) \leftrightarrow (p \wedge \sim q)$
23. (x=0 and y=0) is a sufficient condition for xy = 0.
24. (x=0 and y=0) is a necessary condition for xy = 0.
25. If xy $\neq$ 0, then x = 0 and y = 0.
26. If xy $\neq$ 0, then x = 0 or y = 0.
27. If $A \cap B \cap C = \varnothing$, then the class {A,B and C} is pairwise disjoint.
28. If the class {A,B,C} is pairwise disjoint, then $A \cap B \cap C = \varnothing$.
29. For any sets X, Y and Z, $n(X \cup Y \cup Z) = n(X) + n(Y) + n(Z)$.
30. If B $\subset$ A, then n(A – B) = n(A) – n(B).
31. If B $\subset$ A, then $A^C \subset B^C$.
32. If $(A \cap B) \cup C = C$, then $A \cap B \subset C$.
33. A class that is closed under finite union will also be closed under countable union.
34. A class that is closed under countable union will also be closed under finite union.
35. If $\omega \in \bigcup_{\lambda \in \Lambda} A_\lambda$, then $\omega \in \bigcap_{\lambda \in \Lambda} A_\lambda$.
36. $(A - B)^C = A^C - B^C$.
37. The sequence $\{A_n\}$ where $A_n = [2 + \frac{1}{n}, 5 + \frac{1}{n}]$ is a monotone sequence of sets.
38. $\sum_{j=1}^{n} ar^j = \frac{a(1-r^n)}{1-r}$
39. $Q \vee (R \wedge S)$
$Q \rightarrow S$
$\therefore S$
40. I will pass Stat 117.

-END OF EXAM-