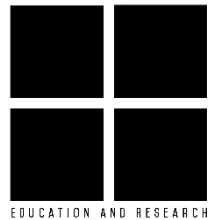
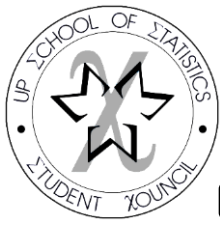


I. FILL IN THE BLANKS.

- The minimal sigma-field containing C_1 denoted by $\sigma(C)$, is defined as a collection of subsets of Ω that satisfies the following:
 - _____
 - _____
 - _____
- The field, denoted by F , is defined as a collection of subsets of Ω that satisfies the following:
 - _____
 - If _____, then _____.
 - If _____, then _____.
- The class $\{A_1, A_2, \dots, A_n\}$ where $A_i \subset A$ ($i=1,2,\dots,n$) is defined to be a partition of set A if and only if it satisfies the following:
 - _____
 - _____
- The limit of a monotone nondecreasing sequence of sets $\{A_n\}$ is _____.
- The limit of a monotone nonincreasing sequence of sets $\{A_n\}$ is _____.
- The steps in proving that statement $P(n)$ is true $\forall \mathbb{Z}^+_n$ by mathematical induction are:
 - Step 1: _____
 - Step 2: _____
 - Step 3: _____
 - Step 4: Conclude that $P(n)$ is true for all positive integer n .
- In proving the statement $p \rightarrow q$,
 - By direct method, we assume that _____ is/are true.
 - By contradiction, we assume that _____ is/are true.
 - By contrapositive, we assume that _____ is/are true.
- In disproving $p \rightarrow q$, we need to show a counterexample for which _____ is/are true.
- In proving $A \subset B$, we need to show that $\omega \in A$ implies _____.
- In proving that A, B and C are pairwise disjoint, we need to show:
 - _____
 - _____
 - _____
- Suppose an urn contains M balls for which K are defective and $M-K$ are non-defective.
 - The number of ordered samples with replacement of size n containing r defectives and $n-r$ non-defectives is _____.
 - The number of ordered samples without replacement of size n containing r defectives and $n-r$ non-defectives is _____.
 - The number of unordered samples without replacement of size n containing r defectives and $n-r$ non-defectives is _____.



12. a) $\sum_{x=0}^n \binom{n}{x} a^{x-1} = \underline{\hspace{2cm}}$. d) $\sum_{i=0}^{20} \binom{20}{i} \frac{3^{i+1}}{2} = \underline{\hspace{2cm}}$.
- b) $\sum_{j=0}^{\infty} \binom{10+j}{j} 0.2^j = \underline{\hspace{2cm}}$. e) $\sum_{j=1}^{100} (3^{2j} + 3j) = \underline{\hspace{2cm}}$.
- c) $\sum_{j=0}^{\infty} \frac{2^j}{j!} j = \underline{\hspace{2cm}}$. f) $\sum_{x=3}^{\infty} \frac{3}{x^2 + x} = \underline{\hspace{2cm}}$.

13. Let Ω = set of real numbers. Let $f(x) = (2x)I_{[-1,1]}(x) + (x-3)I_{(0,+\infty)}(x)$
 a) $f(1) = \underline{\hspace{2cm}}$ b) $f(-3) = \underline{\hspace{2cm}}$.
14. The number of ways of forming 5 committees with 3 members each is $\underline{\hspace{2cm}}$.
15. The number of distinct permutations of the word 'CHARACTER' is $\underline{\hspace{2cm}}$.
16. The number of distinct outcomes when we toss a die 10 times is $\underline{\hspace{2cm}}$.
17. The number of 5-digit numbers that can be formed using the integers 0 to 9 is $\underline{\hspace{2cm}}$.
18. An urn contains 10 balls marked from 1-10. Two balls will be selected using simple random sampling with replacement. Let $\Omega = \{(\omega_1, \omega_2) : \omega_i \in \{1, 2, \dots, 10\}, i = 1, 2\}$. Define A_{ij} = set containing outcomes where the i th ball is selected on the j th draw.
 a) The elements of set A_{11} are $\underline{\hspace{2cm}}$.
 b) The elements of set $A_{11}^c \cap A_{12}$ are $\underline{\hspace{2cm}}$.
 c) The set containing outcomes where the 1st ball is not selected on any of the draws can be represented using the A_{ij} 's as $\underline{\hspace{2cm}}$.

II. TRUE OR FALSE.

- The set of real numbers is equivalent to the set of counting numbers.
- The set of positive integers is equivalent to the set of integers.
- The set of rational numbers is a non-denumerable set.
- The set of all outcomes of tossing a coin 1,000,000 times is a finite set.
- $F(x) = x^2 - 1$ is one-to-one but not an onto function from the set of positive real numbers to the set of all real numbers.
- $G(x) = |x - 1|$ is onto but not one-to-one function from the set of nonnegative integers to the set of nonnegative integers.
- If A is a field, then A is a sigma-field.
- A subset of the set of all real numbers is a Borel set.
- The Borel field is a minimal field.
- A field is closed under finite intersection.
- A field is closed under countable intersection.
- The Borel field is closed under finite union.
- The Borel field is closed under countable intersection.
- $A - B$ and $B - A$ are pairwise disjoint.



15. To disprove the statement $\forall x, P(x)$ is true, it is enough to provide an example for which the proposition $P(x)$ is false for some x in the domain of discourse.
16. To prove the statement $\exists x, P(x)$ is true, it is enough to provide an example for which the proposition $P(x)$ is true for some x in the domain of discourse.
17. The proposition $p \vee \sim p$ is a tautology.
18. The proposition $p \vee p$ is a tautology.
19. The propositions $P \equiv (p \equiv q)$ and $Q \equiv (p \wedge q) \vee (\sim p \wedge \sim q)$ are logically equivalent to each other.
20. The propositions $P \equiv ((p \vee q) \wedge r)$ and $Q \equiv (p \vee q)$ are logically equivalent to each other.
21. $((p \vee q) \wedge r) \rightarrow (p \vee q)$
22. $\sim (p \rightarrow q) \leftrightarrow (p \wedge \sim q)$
23. $(x=0 \text{ and } y=0)$ is a sufficient condition for $xy = 0$.
24. $(x=0 \text{ and } y=0)$ is a necessary condition for $xy = 0$.
25. If $xy \neq 0$, then $x = 0$ and $y = 0$.
26. If $xy \neq 0$, then $x = 0$ or $y = 0$.
27. If $A \cap B \cap C = \emptyset$, then the class $\{A, B \text{ and } C\}$ is pairwise disjoint.
28. If the class $\{A, B, C\}$ is pairwise disjoint, then $A \cap B \cap C = \emptyset$.
29. For any sets X, Y and Z , $n(X \cup Y \cup Z) = n(X) + n(Y) + n(Z)$.
30. If $B \subset A$, then $n(A - B) = n(A) - n(B)$.
31. If $B \subset A$, then $A^c \subset B^c$.
32. If $(A \cap B) \cup C = C$, then $A \cap B \subset C$.
33. A class that is closed under finite union will also be closed under countable union.
34. A class that is closed under countable union will also be closed under finite union.
35. If $\omega \in \bigcup_{\lambda \in \Lambda} A_\lambda$, then $\omega \in \bigcap_{\lambda \in \Lambda} A_\lambda$.
36. $(A - B)^c = A^c - B^c$.
37. The sequence $\{A_n\}$ where $A_n = [2 + \frac{1}{n}, 5 + \frac{1}{n}]$ is a monotone sequence of sets.
38. $\sum_{j=1}^n ar^j = \frac{a(1-r^n)}{1-r}$
39. $Q \vee (R \wedge S)$
 $Q \rightarrow S$
 $\therefore S$
40. I will pass Stat 117.