UP SCHOOL OF STATISTICS STUDENT COUNCIL

EDUCATION & RESEARCH

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S117_LE1_002

Statistics 117 — Mathematics for Statistics

First Long Examination — August 16, 2006

The following definitions and axiom will be useful in answering this exam:

Definition: x is an *even integer* iff there exists an integer n such that x=2n.

Definition: x is an *odd integer* iff there exists an integer n such that x=2n-1.

Definition: Let a and b be integers with $a\neq 0$. We say that *a divides b*, denoted by a|b, iff there exists an integer c such that ac=b.

Definition: The number v is called an *upper bound* of a set S iff for every $x \in S$, $x \leq v$.

Definition: The number b is the *least upper bound* of a set S iff (i) b is an upper bound of S, and, (ii) for every $v \in V$, $b \leq v$ where V = set of all upper bounds of S.

Completeness Axiom: Every set S is a subset of $\Re$ and has an upper bound has a least upper bound. Similarly, every such set S that has lower bound has a greatest lower bound.

Definition: The limit of f(x) as x approaches a is L, denoted by $\lim_{x\to a} f(x) = L$, iff for any **ε > 0**, there exists **δ > 0** such that **0 < |x-a| < δ** implies that **|f(x)-L| < ε**.

I. TRUE OR FALSE. Write True if the statement is always true. Otherwise, write False.

Let p, q and r be propositions.

1. $\sim(p\vee q) \rightarrow \sim q$ true
2. $((p\wedge q) \rightarrow r) \equiv (p\rightarrow(r\vee \sim q))$ true

Let x and y be real numbers.

3. A sufficient condition for $x^2 - 2x + 1 = 0$ is x=1.
4. A necessary condition for $x^2 - 2x + 1 = 0$ is x=1.
5. $x \leq y$ and $x \geq y$ is a contradiction.
6. xy = 0 iff (x=0 or y=0).
7. $xy \neq 0$ iff ($x\neq 0$ or $y\neq 0$).

Let A=set of even integers and B=set of all integers.

8. There exists $x\in A$, for any $y\in B$ such that x=2y.
9. For any $x\in A$, there exists $y\in B$ such that $x\neq 2y$.
10. For any $x\in B$, there exists $y\in B$ such that x=2y.

II. Construct the truth table of the following proposition:

Let p, q and r be propositions,

$$((p \wedge q) \vee \sim r) \leftrightarrow (q \rightarrow r)$$

III. Prove the validity of the following arguments using the rules of inference.

1. $Q \vee (R \wedge S)$
   $\underline{Q \rightarrow S}$
   $\therefore S$

2. If you are young then you are restless and gullible.
   If you are restless or you are gullible then you often make mistakes.
   Therefore, if you are young then you often make mistakes.

3. Let the domain of discourse be the set of all students.
   All intelligent Stat majors will become rich.
   All lazy students are intelligent but will not become rich.
   Therefore, all Stat majors are not lazy.

IV. **PROVING.**

1. Prove by contrapositive: Let x, y and z be real numbers. If $x \leq y$ and $z < 0$ then $xz \geq yz$.
2. Prove by contradiction: If $S=\{x \in \mathbb{Z}: x=2n$ where n is an integer$\}$ then S has no upper bound.
3. Prove by mathematical induction: For every positive integer n, $\sum_{x=1}^{n} \frac{1}{2^x} = 1 - \frac{1}{2^n}$.
4. Prove: There exists an integer y such that $9|y$ and $6|y$.
5. Prove: $\lim_{x \to 0} 10x = 0$.

V. Disprove the following.

1. If x and y are real numbers and $x<y$ then $x^2 \leq y^2$.
2. For any real number x, $x^2 \geq x$.
3. Let p, q and r be propositions. $(p \vee q) \rightarrow (q \wedge r)$.