UP SCHOOL OF STATISTICS STUDENT COUNCIL

EDUCATION & RESEARCH

PHILOS SOPHIA

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Statistics 117 — Mathematics for Statistics

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I. **TRUE OR FALSE.** Write 'True' if the statement is always true; otherwise, write 'False'.

1. If $\Omega=\{1,2,3,4,5\}$ and $\mathcal{A}=\{\Omega, \Phi, \{1\}, \{2,3\}, \{1,2,3\}, \{2,3,4,5\}, \{1,4,5\}, \{4,5\}\}$ then $\mathcal{A}$ is an algebra.
2. If $\mathcal{A}=\{A_1, A_2, A_3\}$ and $\bigcap_{i=1}^{n} A_i = \phi$ then $\mathcal{A}$ is pairwise disjoint.
3. If A and B are sets then $A - B \subset A \cup B$.
4. If $A \subset B$ and $B \subset C$ then $\{A, B - A, C - B\}$ is pairwise disjoint.
5. If $A=\{1,2,3,4,5,\}$ then $\{\{1,5\}, \{3\}, \{4,2\}\}$ is a partition of A.
6. If $\mathcal{A}$ is a field then $\mathcal{A}$ is a sigma-field.
7. The set of all integers is a Borel set.
8. The Borel field contains all the possible subsets that can be formed from the set of real numbers.
9. If $\{A_n\}$ is a monotone nonincreasing sequence of sets then $\bigcup_{i=1}^{\infty} A_i = A_1$.
10. The sequence $\{A_n\}$ where $A_n=[2 + 1/n, 5 ++ 1/n]$ is a monotone sequence of sets.

II. **DEFINITION OF TERMS.**

1. *What are the axioms that define a sigma-field,* $\mathcal{F}$?
2. What are the axioms that define a minimal field containing the class $\mathcal{C}$?
3. What are the axioms that define a partition, $\{A_1, A_2, \dots, A_n\}$ where $A_i \subset A$ for all i, of the set A?
4. State any 3 necessary and sufficient conditions for $A \subset B$.
5. Define a monotone nondecreasing sequence of sets, $\{A_n\}$. What is the limit of this sequence?
6. State all the propositions that you need to prove in order to show by definition that {A, B, C, D} is pairwise disjoint.
7. Fill in the blank. $\mathcal{F}$ Is closed under finite union iff for any $n \in \{2,3,4,\dots\}$, ________________________.
8. Fill in the blanks. The Borel field is the a) _____________________ containing the class $\mathcal{C}$ where b) $\mathcal{C}$ = ___________ and c) Ω = ___________.

III. **IDENTIFY THE GENERALIZED UNION AND INTERSECTION** (where Λ = index set)

1. $A_\lambda = [5 + 2/\lambda, 10 + 2/\lambda]$, $\Lambda = \{1,2,3,4,5\}$
2. $A_\lambda = [5 + 2/\lambda, 10 + 2/\lambda]$, Λ = set of positive integers
3. $A_\lambda = \{\lambda, \lambda-1, \lambda-2,\dots,2,1,0\}$, Λ = set of positive integers
4. $A_\lambda = \{\lambda-1, \lambda, \lambda+1\}$, $\Lambda = \{1,2,\dots,100\}$
5. $A_\lambda = (-1, 1/\lambda)$, $\Lambda = (0,1)$

IV. Suppose a die is tossed 3 times. We can represent an outcome of this experiment by an ordered 3-tuple where the i^{th} coordinate represents the number of dots on the j^{th} toss. Let Ω = set of all possible outcomes = $\{(x,y,z) \mid x,y,z \in \{1,2,3,4,5,6\}\}$. Define A_{ij} = the set containing outcomes where i dots appear on the j^{th} toss, i=1,2,3,4,5,6 and j=1,2,3.

For example, A_{12} = the set containing outcomes where 1 dot appears on the 2^{nd} toss = {(1,1,1), (1,1,2), (1,1,3), (1,1,4), (1,1,5), (1,1,6), (2,1,1), (2,1,2), (2,1,3), (2,1,4), (2,1,5), (2,1,6), (3,1,1), (3,1,2), (3,1,3), (3,1,4), (3,1,5), (3,1,6), (4,1,1), (4,1,2), (4,1,3), (4,1,4), (4,1,5), (4,1,6), (5,1,1), (5,1,2), (5,1,3), (5,1,4), (5,1,5), (5,1,6), (6,1,1), (6,1,2), (6,1,3), (6,1,4), (6,1,5), (6,1,6)}

a) Specify the set $A_{53} \cap A_{41}$ using the roster method.
b) Let B = set containing outcomes where 5 dots appear in all 3 tosses. Express set B as a *composition of the* A_{ij}.
c) Let C = set containing outcomes where 5 dots appear in exactly 1 tosses. Express set C as a composition of the A_{ij}.

V. Let Ω={a,b,c,d,e,f,g} and $\mathcal{C}$ ={{b,c,d,e}, {e,f}}. Construct the minimal sigma-field containing $\mathcal{C}$.

VI. PROVING. Justify each line of your proof.

1. Prove: If $\mathcal{F}$ is closed under the union of 2 sets then $\mathcal{F}$ is closed under finite union.
2. Prove using only the definitions of equality of 2 sets, subset, set operations and rules of inference to prove the following: If $A \subset B$ then $A \cap B = A$.
3. Prove de Morgan's theorem, $\left(\bigcup_{\lambda \in \Delta} A_\lambda\right)^c = \bigcap_{\lambda \in \Delta} A_\lambda^c$.
4. Prove: If $A \subset C$ then $(A \cup B) - C = B - C$.