S117-LE1-002

Statistics 117 | Mathematics for Statistics
First Long Examination | TGCapistrano

I. TRUTH TABLE.

1. Construct the truth table of the following proposition:

$$(p \to r) \leftrightarrow ((\sim p \vee q) \wedge (\sim q \vee r))$$

2. Is $(p \to r)$ logically equivalent to $((\sim p \vee q) \wedge (\sim q \vee r))$? Justify your answer using the truth table you constructed in #1.

II. PROVING THE VALIDITY OF AN ARGUMENT. Construct a formal proof of validity of the following arguments using the rules of interference and/or quantification rules.

1. If you are well-read and you are a good speaker then many people enjoy your company.
If you are a fanatic then you are a good speaker but many people do not enjoy your company.
Therefore, if you are well-read then you are not a fanatic.

2. Let the domain of discourse be the set of all people.
All who do not enjoy puzzles are impatient.
All who enjoy puzzles are good in math and enjoy playing computer.
Ana is not good in Math.
Therefore. Ana is impatient.

III. PROVING. Prove the following theorems. Strictly follow instructions whenever a method of proof has been specified. Always write down what your assumptions are and what you need to show. Provide a justification for each line of your proof.

You may use the axioms in the real number system in Chapter 1 of our notes in justifying the lines of your proof. In addition, you may also use only the following theorems and definitions:

THEOREMS: Let a, b, x, and y be real numbers.

(i) If $xy = 0$ then $x = 0$ or $y = 0$.
(ii) If $xy \neq 0$ then $x \neq 0$ and $y \neq 0$.
(iii) If $xy > 0$ then ($x>0$ and $y>0$) or ($x<0$ and $y<0$)
(iv) If $xy < 0$ then ($x>0$ and $y<0$) or ($x<0$ and $y>0$)
(v) Special Product: $x^2 - y^2 = (x+y)(x-y)$
(vi) If $x = y$ and $ax = b$ then $ay = b$.
(vii) $x + (-y) = x - y$
(viii) $(-1)x = -x$
(ix) $(-x)(-y) = xy$
(x) $x \geq y$ or $x < y$ is a tautology.

DEFINITIONS:

(i) Let a and b be integers where $a \neq 0$. We say that b is divisible by a, denoted by $a|b$ iff $\exists\, c \in \mathbb{Z}$, $ac=b$.
(ii) The absolute value of x, denoted by $|x|$, is defined as $|x|=x$, if $x\geq 0$ and $|x|= -x$ if $x<0$/

In nos. 1 to 4, let x, y, z be real numbers.

1. Prove by direct method: If $x>0$ and $y>0$ then $x + y >0$.
2. Prove by contradiction: If $x>0$ and $y>0$ and $x\geq y$ then $x^2 \geq y^2$. (Note: You may use the theorem proven in #1 in your proof).
3. Prove by contrapositive: Let x, y and z be real numbers. If $x\leq y$ and $z<0$ then $xz\geq yz$
4. Prove by contradiction: If $c>0$ and $|x|\geq c$ then $x\geq c$ or $x\leq -c$.
5. Prove by direct method: Let m, n, o, p be integers where m and n are non-zero. If $m|o$ and $n|p$ then $mn|op$.
6. Prove $\exists\, n \in \mathbb{Z}^+, 7 \mid (3^n + 1)$.
7. Prove by mathematical induction: For any $n \in (2, 3, 4, \ldots), 2 \mid (n^2 - n)$.
8. Prove: $\lim_{x\to 3} 4x = 12$, that is, for any $\varepsilon >0$, there exists $\delta >0$, such that if $|x-3|<\delta$ then $|4x-12|<\varepsilon$.

IV. DISPROVING.

1. Let p. q. and r and s be propositions. Show that the following argument is not valid

$$\begin{array}{l} (p \wedge q) \rightarrow r \\ \underline{s \qquad\qquad} \\ \therefore ((p \vee q) \wedge s) \rightarrow r \end{array}$$

2. Disprove. Let x and y be real numbers. If $x^2 > y^2$ then $x>y$.