UP SCHOOL OF STATISTICS STUDENT COUNCIL

Education and Research

erho.weebly.com erhomyhero@gmail.com /erhoismyhero @erhomyhero

S117-LE2-001

Statistics 117 Mathematics for Statistics

Second Long Examination TGCapistrano

I. TRUE or FALSE. Write “True” if the statement is always true, otherwise, write “False”.
Let A, B and C be sets.

1. The field is closed under the operation of set difference.
2. Some subsets of the set of real numbers are not Borel sets.
3. If $A-(B\cap C)=\varnothing$ then $A\subset (B\cap C)$.
4. If $A\cap B\cap C=\varnothing$ then A, B and C are pairwise disjoint.
5. If $\{A, B-A, C-(A\cup B)\}$ is a partition of $A\cup B\cup C$.
6. $\mathscr{C}$ is closed under the intersection of two sets if and only if it is closed under finite intersection.
7. If $A\subset B$ then $C=\{\varnothing,\Omega,A,A^c,B,B^c,B-A,(B-A)^c\}$ is a field.
8. $\mathscr{C}=\{\varnothing,\Omega,A,A^c\}$ is a sigma-field.
9. A sigma-field is closed under generalized union.
10. The set of all positive rational numbers is a Borel set.

II. Let $\Omega=$ set of reals. For the following sequences of $\{A_n\}$

a) Write ‘M’ if it is a monotone sequence; otherwise, write ‘N’
b) If it is a monotone sequence, identify the limit of the sequence. If it is not a monotone sequence, identify the countable union and countable intersection.

1. $A_n=\{0,1,2,\ldots,n\}$
2. $A_n=\left\{5-\frac{1}{n},8-\frac{1}{n}\right\}$
3. $A_n=\left\{-\frac{2}{n},\frac{2}{n}\right\}$
4. $A_n=\{5-2n,8+2n\}$
5. $A_n=\{n-1,n,n+1\}$

III. State the complete definition of the following terms by filling in the blanks.

1. The class $\mathscr{F}$ is a filed iff _________________.
2. The class $\mathscr{F}(\mathscr{C})$ is the minima sigma field containing the class $\mathscr{C}$ iff _________________.
3. The Borel field, denoted by is ________ where $\Omega=$ ________ and $\mathscr{C}$ = _________.
4. The class $\{A_1,A_2,\ldots,A_n\}$ is pairwise disjoint iff ______________.
5. $\omega\in\bigcup_{\lambda=\Delta}A_\lambda$ iff _____________.
6. The class $\mathscr{C}$ is closed under finite union iff ___________.
7. The class $\mathscr{C}$ is closed under countable intersection iff ______________.

IV. Define $\Omega = \{a, b, c, d, e, f\}$

$\mathscr{C} = \{\{a\}, \{a, c\}, \{a, d, e\}\}$

1. Construct the minimal sigma-field containing $\mathscr{C}$.
2. Specify by roster method a partition of Ω.

V. A. Suppose a newly-married couple plans to have four children.

Define $\Omega = \{(\omega_1, \omega_2, \omega_3, \omega_4) : \omega_1 =$ sex of the ith child $\in \{G, B\}, i = 1, 2, 3, 4\}$

$G_i =$ set of elements of Ω where the ith child is a girl, $i = 1, 2, 3, 4$

Express the following sets as a composition of the G_i 's

1. A = the set containing elements of Ω where only the second child is a boy
2. B = the set containing elements of Ω where there is at least one girl
3. C = the set containing elements of Ω where there is exactly 1 boy

B. Suppose a die is tossed four times.

Define $\Omega = \{(\omega_1, \omega_2, \omega_3, \omega_4) : \omega_1 =$ no. of dots on the ith toss $\in \{1, 2, 3, 4, 5, 6\}, i = 1, 2, 3, 4\}$

$A_j =$ set of elements of Ω where the j dots come up on the ith toss, i=1,2,3,4 and j=1,2,3,4,5,6

Express the following sets as a composition of A_j

1. A = the set containing elements of Ω where 3 dots come up in all 4 tosses
2. B = the set containing elements of Ω where 3 dots come up in at least one of the 4 tosses

VI. Prove the following statements. Provide a justification for each line of your proof.

1. Prove by mathematical induction: If $\mathscr{C}$ is closed under the union of 2 sets then it is closed under finite union.
2. Prove using the definition of equality of two sets and without using the properties of set operations:

   If A and B are sets then $A \cup (A \cap B) = A$

3. Prove de Morgan's Law: $\left(\bigcap_{\lambda = \Delta} A_\lambda \right)^c = \bigcap_{\lambda = \Delta} A_\lambda^{\ c}$
4. Prove using the properties of set operations:

   If A and B are sets then $(A - B) \cup (B - A) = (A \cup B) - (A \cap B)$

5. Prove using the definition of a sigma-field:

   If $\mathscr{F}$ is a sigma-field then it is closed under countable intersection.

VII. Disprove the following statements by providing a counterexample. Use $\Omega = \{a, b, c, d\}$. Let A, B and C be sets.

1. If $A \cup B = A \cup C$ then $B = C$.
2. If $B = A \cup C$ then $A = B - C$.