S117-LE3-001
Statistics 117 Mathematics for Statistics
Third Long Exam Second Semester, AY 2013 -2014

I. Determine if the following are function or relation. If a function, state whether one-to-one, onto or neither.

1. A = All reals; B = All reals; $f(x) = x + 10, x \in A$
2. A = Set of positive integers; B= Set of real numbers; $f(n) = n + \frac{1}{n}, n \in A$
3. A = Set of positive integers; B = Set of positive integers; $f(x) = [x] + [x-1], x \in A$

II. Proving

1. Prove: $I_{A\cup B\cup C}(\omega) = I_A(\omega) + I_B(\omega) + I_C(\omega) - I_{A\cap B}(\omega) - I_{A\cap C}(\omega) - I_{B\cap C}(\omega) + I_{A\cap B\cap C}(\omega)$
2. Prove: $I_{A\Delta B}(\omega) = I_A(\omega) + I_B(\omega) - 2I_{A\cap B}(\omega)$

III. Evaluate the following.

1. $\sum_{x=10}^{100}(4x-100)^2$
2. $\sum_{x=0}^{\infty}\frac{1}{5^x x!}$
3. $\sum_{x=2}^{100} 4^{x+1}$
4. $\sum_{x=0}^{\infty}\binom{4+x}{x}0.25^x$
5. $\sum_{x=1}^{\infty} 0.5^{3x}$
6. $(x^{1/3} - y)^5$
7. $\sum_{x=0}^{20}\binom{20}{x}(x)(10^x)$
8. $\sum_{x=0}^{n} x(x-1)0.7^x 0.3^{n-x}$

IV. Counting Techniques

1. A computer password consists of a letter of the alphabet followed by 3 or 4 digits. Find (*a*) the total number of passwords that can be created, and (*b*) the number of passwords in which no digit repeats.
2. Find the probability p_n that a randomly assembled group of n people includes at least 2 people with the same birthday (day of the year).
3. In a dormitory, there are 12 students who take an art course (A), 20 who take a biology course (B), 20 who take a chemistry course (C), and 8 who take a drama course (D). There are 5 students who take both A and B, 7 students who take both A and C, 4 students who take both A and D, 16 students who take both B and C, 4 students who take both B and D, and 3 students who take both C and D. There are 3 who take A, B, and C; 2 who take A, B, and D; 2 who take B, C and D; 3 who take A, C, and D. Finally, there are 2 in all four courses. It is also known that there are 71 students in the dormitory who have not signed up for any of these courses. Find the total number of students in the dormitory.
4. Disregarding order within a box, find the number of ways of packing: (*a*) 12 distinct books in 3 distinct boxes so that 3 books are in box 1, 4 books are in box 2, and 5 books are in box 3; (*b*) 12 distinct books in 3 distinct boxes so that 1 of them has 3 books, another has 4 books, and yet another has 5 books; (*c*) 12 distinct books in 3 identical boxes, if they are to contain 3, 4, and 5 books; (*d*) 12 distinct books in 3 distinct boxes, each to contain 4 books; and (*e*) 12 distinct books in 3 identical boxes, each to contain 4 books.