S117-LE3-001

Statistics 117   Mathematics for Statistics
Third Long Examination   TGCapistrano

I.   Write "True" if the statement is always true, otherwise, write "False".

1.   The collection of odd numbers is equivalent to the collection of rational numbers.
2.   The set of all real numbers is equivalent to (0, 1).
3.   The set of positive real numbers is equivalent to the set of positive integers.
4. $A = \{y \in \mathbb{R} : y = e^x$ where x is a real number$\}$ is a countably infinite collection.
5.   The set of positive integers less than 10000^{10000} is a finite collection.
6.   The closed interval;, [1, 10] is a noondenumerable set.
7.   If A={2,4,6,8,10} and B={1,2,3,4,5,6} then f={(2,1),(4,2),(6,3),(8,4),(10,5)} is a function with domain A and counterdomain B.
8.   The function f: $\mathbb{R}^+ \to \mathbb{R}$ where $f(x) = x^2 + 1$ for any $x \in \mathbb{R}^+$ is a one-to-one function.
9.   The function f: $\mathbb{R}^+ \to \mathbb{R}$ where $f(x) = \log(x)$ is an onto function

10. $\left(\sum_{i=1}^{n} a_i \right)^2 = \sum_{i=1}^{n} a_i^2 + \sum_{i=1}^{n} \sum_{\substack{j=i \\ i<j}}^{n} a_i a_j$

II.   Answer the following:

1.   Give the exact value of the following expressions:

a) $\dbinom{1000}{998} =$

b) $\dbinom{-1.5}{3} =$

c) Let $f(x) = x^2 \big|_{[1,10]} (x) + (3x^3 - 2x^2 + 4x - 1) \big|_{[3,5]} (x)$. What is f(10)?

d) $\sum_{x=1}^{3} \sum_{y=2}^{3} xy =$

2.   PROVE. For any positive integers n and k where k≤n. $\dbinom{n+1}{k} - \dbinom{n}{k} = \dbinom{n}{k-1}$

III.   EVALUATE THE FOLLOWING SUMS: (Show your solution. Just leave your answer in formula form. Answer must not involve the summation operator nor any letters.)

1. $\sum_{j=0}^{100} 2^{3j+1}$
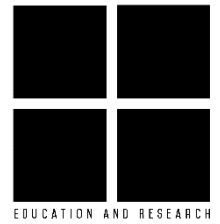
2. $\displaystyle\sum_{j=0}^{\infty}\binom{j+15}{j}(0.2)^{j-2}$

3. $\displaystyle\sum_{j=150}^{300}\frac{j(j^2+4)}{2}$

4. $\displaystyle\sum_{j=0}^{19}\binom{20}{j}2^{18-j}$

5. $\displaystyle\sum_{j=2}^{\infty}\frac{j(j-1)2^j}{j!}$

6. $\displaystyle\sum_{j=1}^{100}\left(\frac{1}{j(j+1)}-j^2\right)$

7. $\displaystyle\sum_{i=1}^{250}\sum_{\substack{j=1\\i<j<k}}^{250}\sum_{k=1}^{250}57$

IV.   ANSWER THE FOLLOWING: (Except for #12, present your answers in formula form with all the appropriate values plugged in.)

1. An organization has 120 members.
   a) How many possible ways can we partition the members of this organization into 10 committees with 12 members each?
   b) How many possible ways can we select 5 members of the organization to form a committee of size 5?
   c) How many ways can we arrange all the members of the organization in a single line?
2. The Ajax Phone Company offers three basic models of telephone (princess, desk, wall). Each model with either touch-tone, or automatic dialing. How many variations of phone does the Ajax company have?
3. How many ways can we arrange 5 boys and 10 girls in a line where all the 5 boys are always together?
4. How many ways can we arrange the letters of the word "STATISTICS"?
5. How many ways can we arrange the letters of the word "UNIVERSAL" if the first two letters must both be consonants while the last two letters are vowels?
6. How many ways can a family of eight be seated at a circular table?
7. Consider the set of digits {3,4,5,6,7,8,9}.
   a. How many 4-digit numbers can be forms from this set of digits can be used only once?
   b. How many of the 4-git numbers in (a) are even?
   c. How many of the 4-git numbers in (a) where exactly of the digits are even numbers?
   d. How many of 4-digit numbers in (a) are greater than 4327?
8. A poker hand consists of 5 cards from a standard decj of 52 cards.
   a. How many possible poker hands are there if we ignore the order in which the cards are dealt?
   b. How many among the poker hands in (a) consist of exactly 2 hearts?
   c. How many among the poker hands in (a) consist of 2 hearts, 2 spades, 1 club?
   d. How many among the poker hands in (a) consist of 5 cards that are all of the same suit?
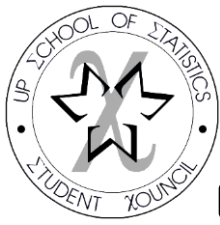
9. In the PCSO Lotto 6/42, a 6-number combination is drawn form a lot of numbers from 1 to 42 at random. To win a prize, at least three of the player's chosen numbers must match with those of the six winning number.
   a. How many 6-number combinations are possible in this lotto?
   b. How many 6-number combinations in (a) contain exactly 3 of the winning numbers?
   c. How many 6-number combinations in (a) contain at least 3 of the winning numbers?

10. Suppose a precinct consists of 150 voters, where 80 of them are less than 30 years old. Suppose a sample of 25 voters will be selected in this precinct.
    a. How many ordered samples without replacement are there?
    b. How many of the ordered samples in (a) include exactly 10 voters who are less than 30 years old?
    c. How many of the ordered samples in (a) include at least 10 voters who are less than 30 years old?

11. Suppose we roll a die 10 times and we record the number of dots that come up on each roll.
    a. How many possible outcomes are there if an outcome is represented by an ordered 10-tuple?
    b. How many among the outcomes in (a) where the first 2 rolls both result in a number less than 5?
    c. How many among the outcomes in (a) where exactly 2 rolls both result in a number less than 5?

12. (INSTRUCTION: Define the basic sets. In each item, express the set whose cardinality you are interested in terms of the defined basic sets. Write the formula/s used to compute for the cardinality. Compute for the cardinality using the formula and the given information.)

    A sample of 250 voters was taken from the 3rd district of Quezon City. Each one was asked whom they voted for councilors in the last elections. The survey showed that 102 voted for Candidate A, 103 voted for Candidate B, and 95 voted for Candidate C. There were 67 who voted for A but not B; 62 who voted for A but not C; and, 567, who voted for B and not C. Only 15 voted for all 3 candidate, A, B and C.
    a. How many did not vote for Candidate B?
    b. How many voted for both Candidates A and C?
    c. How many voted for Candidate B but not A?
    d. How many did not vote for any one of the 3 candidates?