1. a. State the definition of the $n$-th order linear ordinary differential equation.
   b. Classify each of the following ordinary differential equation as linear or nonlinear. Also, determine the order of the equation:
      i. $x^3 y'' + x^2 y'' + xy' + y = x^4$
      ii. $e^y + t \cos y' = 0$

   where $y' = \frac{dy}{dx}$, $y'' = \frac{d^2y}{dx^2}$, $y''' = \frac{d^3y}{dx^3}$, $\dot{y} = \frac{dy}{dt}$.

2. Let us consider the following 1st order linear ordinary differential equation: $\dot{y} + q(t)y = r(t)$, where $q(t)$ and $r(t)$ are continuous for some interval $0 \leq t \leq b$, and $\dot{y} = \frac{dy}{dt}$. Prove the following propositions:
   a. A solution of $\dot{y} + q(t)y = r(t)$ with initial condition: $y(0) = y_o$, is given by
      $$y(t) = \frac{1}{Q(t)}\left(y_o + \int_0^t r(\tau)Q(\tau)d\tau\right),$$
      where $Q(t) := \exp\left(\int_0^t q(\tau)d\tau\right)$.
   b. The solution of $\dot{y} + q(t)y = r(t)$ is unique for the initial condition.

3. Initially, Tank I contains 200 gallons of salt brine with a concentration of 1 pound per gallon, and Tank II contains 100 gallons of salt brine with a concentration of 2 pounds per gallon. Liquid is pumped from Tank I into Tank II at a rate of 3 gallons per minute, and liquid is pumped from Tank II into Tank I at a rate of 2 gallons per minute. The tanks are kept well stirred. What is the concentration in Tank I after 100 minutes?

4. Solve the following first order ordinary differential equations:
   a. $\dot{y} + (\tan t)y = \sec t$, $y(\pi/4) = 0$
   b. $x \frac{dy}{dx} + y - x^3 y^2 = 0$
   c. $\frac{dy}{dx} = \frac{2y}{x}$
   d. $(3t^2 - y^2)dt + (ty - t^3 y^{-1})dy = 0$